A TWO-LEVEL UTILITY FUNCTION AND A STEPPED SUPPLY FUNCTION IN A GENERAL EQUILIBRIUM MODEL OF TRADE

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I. INTRODUCTION

THE Heckscher-Ohlin model assumes constant returns to scale, factor mobility across sectors, perfect competition in the domestic and foreign markets and no dichotomy between domestically produced and imported goods. In this framework, resource allocation is determined by factor rentals being equalized across sectors. Clearly, this implies that the model is long-run in nature and comparative advantage in this long-run equilibrium determines the optimal pattern of trade. The significance of this in terms of multi-sectoral model building can be seen in the context of an important contribution by Samuelson [11]. He showed that the assumptions of the Heckscher-Ohlin model, together with free trade in a situation in which there are n goods and m factors of production, n being greater than m, resulted in a solution in which only m goods would be produced at most. Since the number of factors in most models is usually small, this would mean that the country would find it profitable to produce only a few goods. This has been termed the linearity or specialization problem.

Empirical studies on trade and development in a multi-sectoral framework have tried to overcome the specialization problem by relaxing some of the assumptions of the Heckscher-Ohlin model. Chenery and Raduchel [5], De Melo [7] and Ali [1] [2] introduce increasing cost industries while maintaining factor mobility. In contrast, Evans [8] and Taylor and Black [15] introduce capital specificity to increase the number of factors of production. Staelin [13] assumes that the domestic sector of the economy is noncompetitive and introduces various rules of markup pricing. Evans [9] and Ali [3] draw a distinction between domestically produced and imported goods with finite elasticity of substitution.

Clearly, in a multi-sectoral model in which the number of commodities exceeds the number of primary factors of production, some assumptions of the Heckscher-Ohlin model will have to be dropped if extreme solutions which are not in keeping with reality are to be avoided. However, factor mobility and unconstrained trade activity so essential to the concept of comparative advantage must be maintained. It is in this spirit that the model used in this paper has been developed.

This is a revised version of parts of a doctoral dissertation submitted to the Department of Political Economy, the Johns Hopkins University. I am grateful to Professors Bela Balassa and Trent Bertrand for helpful comments. They are in no way responsible for the remaining errors.

Though many demonstration planning models have been developed for India [4] [14], with the exception of Weisskopf's study [17], they have been singularly lacking in the incorporation of the foreign trade sector. Exports are exogenously determined while upper bounds are put on imports. In Weisskopf's study, the volume and composition of imports were endogenously determined. The shortcoming of the demonstration planning models for India resulted from the inability to overcome the specialization problem.

In this paper, an attempt is made to strengthen the logical foundations of the demonstration planning models thereby improving the empirical validity of the results of comparative advantage that follow. The specialization problem is sought to be overcome both from the demand and supply sides. The attempt here is to move away from the piecemeal approaches used by Ali [2] [3] to a more complete model.

II. THE THEORETICAL FRAMEWORK

A. *Utility Function*

The first point of departure from the traditional Heckscher-Ohlin model is the introduction of the two-level utility function. In traditional trade theory, imported and domestically produced goods are assumed to be infinitely substitutable. In contrast, within each commodity group we distinguish domestically produced and imported commodities and allow for imperfect substitution between differentiated products which are produced at home and abroad. In this sense, our model is more general and includes the traditional framework as a special case. The assumption of weak separability is made to keep the model tractable. It implies that the choice between imported and domestically produced good within a commodity group is independent of the prices of all other commodities concerned. An examination of the relevant first order conditions shows this.[1] Thus, we assume that the utility function is given by

$$U=U(C_{1d},\ C_{2d},\ \cdots,\ C_{nd},\ C_{1m},\ C_{2m},\ \cdots,\ C_{nm}), \tag{1}$$

where C_{id} and C_{im} represent consumption goods produced at home and imported, respectively. Given the assumption of weak separability, from Uzawa and Goldman [16] and Sato [12], it can be shown that the utility function can be written as

$$U=U[C^1(C_{1d},\ C_{1m}),\ C^2(C_{2d},\ C_{2m}),\ \cdots,\ C^n(C_{nd},\ C_{nm})], \tag{2}$$

where $C^i(C_{id}, C_{im})$ is a quantum index of all elements belonging to the ith bundle. This function has two levels. The lower level is concerned with the function C^i while the upper level deals with the global function built up from lower level functions. As shown by Ali [3], the assumption of CES type relationship for the lower level functions and a Cobb-Douglas type relationship for the global function results in the following specification:

[1] See R. G. Gregory, "United States Imports and Internal Pressure of Demand: 1948–68," *American Economic Review* Vol. 61, No. 1 (March 1971). A similar distinction between domestically produced and imported commodities is made.

$$U=\prod_{i=1}^{n}[\{B_iC_{id}^{-\rho_i}+(1-B_i)C_{im}^{-\rho_i}\}^{-1/\rho_i}]^{\delta_i}, \qquad (3)$$

where $\sum_{i=1}^{n}\delta_i=1$.

The Allen-Uzawa partial elasticity of substitution σ^*_{ij} for the above takes the following values:

$$\sigma^*_{ij}=\begin{cases} 1+(1/\delta_S)(\sigma^*_S-1), \text{ if } i \text{ and } j \text{ belong to the same commodity} \\ \text{bundle; } s, l, \text{ if } i \text{ and } j \text{ belong to separate commodity bundles.}\end{cases}$$

An implication of the two level utility function specified in (3) is that the ratio of domestically produced to imported consumption good is determined by the ratio of domestic to imported price. Thus from the demand side, the specialization problem would be alleviated.

B. *Production Relations*

The second departure from the traditional trade model is the introduction of the stepped supply function. The rationale for it goes back to Weisskopf's results for the import substituting sectors in India [17]. He found that within a sector, the marginal cost to save a unit of foreign exchange varies with the particular commodity produced. A country will import substitute completely in that good within a sector in which it has the lowest cost before moving on to import substitute in the good with the next lowest marginal cost. In that sense, the import-substituting sectors reflect rising marginal cost. In Ali [2], this finding was formalized in a production function framework by attributing differences in production costs within a sector to differences in efficiency. This led to the following production relation:

$$X=(Ae^{-t(X-\bar{X})})K^{\alpha}L^{\beta}, \qquad (4)$$

where X is output, K and L are capital and labor inputs respectively and the parameter t captures the effect of declining efficiency as output increases. $\bar{X}$ is total sectoral output in the base period. Equation (4) implies that as output increases efficiency declines. This is the central idea behind the stepped supply function.

Assuming $\alpha+\beta$ to add up to unity and given a wage rate w and a rental c, we can derive the cost function which yields the following marginal cost equation.

$$\frac{dC}{dX}=A^{-1}e^{t(X-\bar{X})}(1+tX)w^{\beta}c^{\alpha}\alpha^{-\alpha}\beta^{-\beta}. \qquad (5)$$

Equation (5) is the general form of the stepped supply function and it slopes upwards because the efficiency term of the production function monotonically declines. It should be noted that equation (4) is valid only for the import-substituting sectors. For the other sectors t is zero.

In general, a production relation with labor L, capital K, land T, and intermediate inputs I would be of the following form:

$$X\leq g[f(K, L, T), I]. \qquad (6)$$

As is common in the trade literature [6], we shall make the simplifying assumption that the elasticity of substitution between material inputs and value added $f(K, L, T)$ is zero, material inputs being used in fixed proportion to output, $I=aX$.

The production relations in keeping with our discussion of the stepped supply function will be specified to be

$$X_i \leq A_i K_i^{\alpha_i} L_i^{\beta_i} T_i^{\gamma_i}, \tag{7}$$

$$X_i \leq A_i^{-t_i(X_i-\bar{X}_i)} K_i^{\alpha_i} L_i^{\beta_i}, \tag{8}$$

$$X_i \leq A_i K_i^{\alpha_i} L_i^{\beta_i}, \tag{9}$$

$$X_i = \frac{X_{ji}}{a_{ji}}. \tag{10}$$

The inequality (7) represents the production relations for the agricultural sectors, (8) does so for the import-substituting sectors and (9) represents the export sectors. X_{ji} is the amount of good j used in the production of good i and a_{ji} is the fixed input-output coefficient for the use of good j in the production of good i.

C. *Commodity Balance Constraints*

The distinction made between domestically produced and imported consumption goods implies that the specification of the commodity balance constraints used in this paper will have to be different from those used in existing planning models. The domestically produced consumer good C_{id} cannot be imported but at the same time the intermediate input and exogenous demand from a sector i could be either domestically produced or imported. These restrictions lead to the following commodity balance constraints:

$$X_i - I_i - C_{id} - E_i \geq 0, \tag{11}$$

$$I_i + M_i - \sum_{j=1}^{n} a_{ij} X_j - \bar{Z}_i \geq 0. \tag{12}$$

Constraint (11) tells us that gross output X_i must be greater than consumption C_{id}, intermediate input and exogenous demand I_i and exports E_i. Constraint (12) implies that the intermediate input $\sum_{j=1}^{n} a_{ij} X_j$ and exogenous demand $\bar{Z}_i$ could be either domestically produced $I_i>0$ or imported $M_i>0$. The infinite elasticity of substitution between domestically produced intermediate input and imported input is the assumption that has been traditionally made in the trade literature.

D. *Primary Factors of Production*

We assume that there are three primary factors of production, namely, capital, labor and land, the supplies of which are all fixed. While capital and labor are mobile across all the sectors, land is assumed to be sector specific and is confined to the agricultural sectors. The latter assumption is justified on the ground that the kind of land needed in each agricultural sector is different and hence not substitutable.

E. *Trade Relations*

The small country assumption is made for imports and nontraditional exports. A downward sloping export demand function is assumed for the traditional export items.

III. THE MODEL AND ITS IMPLICATIONS

A. *Notation*

Endogenous Variables

C_{id} : consumption of domestically produced good i.
C_{im} : consumption of imported good i.
X_i : production of good i.
I_i : intermediate input and exogenous demand for good i met from domestic sources.
M_i : import of good i to meet intermediate input and exogenous demands.
E_i : exports of good i.
K_i : capital employed in the production of good i.
L_i : labor employed in the production of good i.
T_i : land employed in the production of good i.

Shadow Prices

π_i : shadow price associated with commodity balance constraint (11).
π^*_i : shadow price associated with the commodity balance constraint (12).
π_K, π_L, π_{ti} : shadow price of capital, labor, and land, respectively.
π_f : shadow price of foreign exchange.
π_{vi} : shadow price associated with the production relations.
π_{qi} : shadow price of import quota.

Exogenous Variables

$\overline{Z}$: exogenous demand from sector i.
$\overline{K}$: total capital stock.
$\overline{L}$: total labor stock.
$\overline{T}_i$: total land stock in agricultural sector i.
$\overline{D}_i$: maximum foreign exchange deficit.
P^*_i : import price of good i.
$\overline{M}_i$: import quota for sector i.

Parameters

B_i, ρ_i, δ_i : consumption parameters.
a_{ij} : input coefficient of commodity i in sector j.
α_i, β_i, γ_i : share of capital, labor, and land in the production function.
t_i : shift term of the efficiency parameter in the production function of the import-substituting sector i.
c_i : intercept of the export demand function.
e_i : slope of the export demand function.

B. *Primal Form of the Model*

$$\text{Maximize } U=\prod_{i=1}^{n}[\{B_iC_{id}^{-\rho_i}+(1-B_i)C_{im}^{-\rho_i}\}^{-1/\rho_i}]^{\delta_i}$$

subject to

(1) commodity balance constraints

$$X_i-I_i-C_{id}-E_i\geq 0 \qquad (i=1,\cdots,n),$$

$$I_i+M_i-\sum_{j=1}^{n}a_{ij}X_j-\overline{Z}_i\geq 0 \qquad (i=1,\cdots,n).$$

(2) factor supply constraints

$$\overline{K}-\sum_{i=1}^{n}K_i\geq 0,$$

$$\overline{L}-\sum_{i=1}^{n}L_i\geq 0,$$

$$\overline{T}_j-T_j\geq 0 \qquad (j=1,\cdots,m).$$

(3) foreign exchange constraint

$$\overline{D}+\sum_{i=1}^{n}(c_i-e_iE_i)E_i-\sum_{i=1}^{n}P^*_i(M_i+C_{im})\geq 0,$$

c_i is non zero for the traditional export sectors.

(4) production relations

$$A_ie^{-t_i(X_i-\overline{X}_i)}K_i^{\alpha_i}L_i^{\beta_i}T_i^{\gamma_i}-X_i\geq 0,$$

t_i is non zero for the import-substituting sectors only,

γ_i is zero for the nonagricultural sectors.

(5) import quotas

$$\overline{M}_i-C_{im}-M_i\geq 0.$$

The solution to the problem can be characterized by the first order conditions obtained from the Lagrangean,

$$\begin{aligned}L=&\prod_{i=1}^{n}[\{B_iC_{id}^{-\rho_i}+(1-B_i)C_{im}^{-\rho_i}\}^{-1/\rho_i}]^{\delta_i}+\sum_{i=1}^{n}\pi_i(X_i-I_i-C_{id}-E_i)\\&+\sum_{i=1}^{n}\pi^*_i(I_i+M_i-\sum_{j=1}^{n}a_{ij}X_j-\overline{Z}_i)+\pi_K(\overline{K}-\sum_{i=1}^{n}K_i)\\&+\pi_L(\overline{L}-\sum_{i=1}^{n}L_i)+\sum_{i=1}^{j}\pi_{ti}(\overline{T}_i-T_i)\\&+\pi_f\{\overline{D}+\sum_{i=1}^{n}(c_i-e_iE_i)E_i-\sum_{i=1}^{n}P^*_i(M_i+C_{im})\}\\&+\sum_{i=1}^{n}\pi_{vi}\{A_ie^{-t_i(X_i-\overline{X}_i)}K_i^{\alpha_i}L_i^{\beta_i}T_i^{\gamma_i}-X_i\}\\&+\sum_{i=1}^{n}\pi_{qi}(\overline{M}_i-C_{im}-M_i).\end{aligned}$$

The first order conditions are

$$\frac{\partial L}{\partial C_{id}}=\frac{\delta_i B_i C_{id}{}^{-(\rho_i+1)}U}{[B_i C_{id}{}^{-\rho_i}+(1-B_i)C_{im}{}^{-\rho_i}]}-\pi_i \leq 0\,, \tag{i}$$

with equality if $C_{id}>0$.

$$\frac{\partial L}{\partial C_{im}}=\frac{\delta_i(1-B_i)C_{im}{}^{-(\rho_i+1)}U}{[B_i C_{id}{}^{-\rho_i}+(1-B_i)C_{im}{}^{-\rho_i}]}-\pi_f P^*{}_i-\pi_{qi} \leq 0\,, \tag{ii}$$

with equality if $C_{im}>0$.

$$\frac{\partial L}{\partial X_i}=\pi_i-\sum_{j=1}^{n}\pi^*{}_j a_{ji}-\pi_{vi}A_i e^{-t_i(X_i-\bar{X}_i)}K_i{}^{\alpha_i}L_i{}^{\beta_i}T_i{}^{\gamma_i}t_i-\pi_{vi} \leq 0\,, \tag{iii}$$

with equality if $X_i>0$.

$$\frac{\partial L}{\partial I_i}=-\pi_i+\pi^*{}_i \leq 0\,, \tag{iv}$$

with equality if $I_i>0$.

$$\frac{\partial L}{\partial M_i}=\pi^*{}_i-\pi_f P^*{}_i-\pi_{qi} \leq 0\,, \tag{v}$$

with equality if $M_i>0$.

$$\frac{\partial L}{\partial E_i}=-\pi_i+\pi_f(c_i-2e_iE_i) \leq 0\,, \tag{vi}$$

with equality if $E_i>0$.

$$\frac{\partial L}{\partial K_i}=-\pi_K+\alpha_i\pi_{vi}A_i\, e^{-t_i(X_i-\bar{X}_i)}K_i{}^{\alpha_i-1}L_i{}^{\beta_i}T_i{}^{\gamma_i} \leq 0\,, \tag{vii}$$

with equality if $K_i>0$.

$$\frac{\partial L}{\partial L_i}=-\pi_L+\beta_i\pi_{vi}A_i\, e^{-t_i(X_i-\bar{X}_i)}K_i{}^{\alpha_i}L_i{}^{\beta_i-1}T_i{}^{\gamma_i} \leq 0\,, \tag{viii}$$

with equality if $L_i>0$.

$$\frac{\partial L}{\partial T_i}=-\pi_{ti}+\gamma_i\pi_{vi}A_i\, e^{-t_i(X_i-\bar{X}_i)}K_i{}^{\alpha_i}L_i{}^{\beta_i}T_i{}^{\gamma_i-1} \leq 0\,, \tag{ix}$$

with equality if $T_i>0$.

Assuming $\pi_{vi}>0$ implies that

$$X_i=A_i\, e^{-t_i(X_i-\bar{X}_i)}K_i{}^{\alpha}L_i{}^{\beta_i}T_i{}^{\gamma_i}\,, \tag{x}$$

substituting (x) in (iii), (vii), (viii), and (ix) we get

$$\frac{\partial L}{\partial X_i}=\pi_i-\sum_{j=1}^{n}\pi_j a_{ji}-\pi_{vi}(1+t_iX_i) \leq 0\,, \tag{iii$'$}$$

with equality if $X_i>0$.

$$\frac{\partial L}{\partial K_i}=-\pi_K+\pi_{vi}\alpha_i\frac{X_i}{K_i} \leq 0\,, \tag{vii$'$}$$

with equality if $K_i>0$.

$$\frac{\partial L}{\partial L_i} = -\pi_L + \pi_{vi}\beta_i \frac{X_i}{L_i} \leq 0 , \qquad \text{(viii}')$$

with equality if $L_i>0$.

$$\frac{\partial L}{\partial T_i} = -\pi_{ti} + \pi_{vi}\gamma_i \frac{X_i}{T_i} \leq 0 , \qquad \text{(ix}')$$

with equality if $T_i>0$.

C. *Economic Interpretation of the Optimality Conditions*

Assuming that $C_{id}>0$ and $C_{im}>0$, conditions (i) and (ii) imply that the ratio of consumption of domestically produced to imported goods is determined by the ratio of domestic to foreign prices including the premium on quotas. Thus given the modelling assumptions, the consumption bundle *i* will include domestic and imported components thereby circumventing the specialization problem.

On the production side, condition (iii′) implies that when output is positive, the output level is determined at the point where price equals marginal cost, $\sum_{j=1}^{n} a_{ji}\pi^*_j$ is the intermediate input cost and $\pi_{vi}(1+t_iX_i)$ is the factor cost. Thus given π^*_j and π_{vi}, condition (iii′) states that the shadow price of the domestically produced good would rise with increased output of the good *i*. This condition captures the essence of the stepped supply function.

Conditions (iv) and (v) imply that intermediate inputs and goods to meet exogenous demand should be imported if domestic price exceeds the foreign price plus the premium on quotas. Combining conditions (iii′), (iv) and (v), we get

$$\sum_{j=1}^{n} a_{ji}\pi^*_j + \pi_{vi}(1+t_iX_i) \leq \pi_f P^*_i + \pi_{qi} ,$$

with equality if $M_i>0$.

This condition tells us that as long as domestic marginal costs are lower than the imported price of intermediate inputs, it is profitable to use domestically produced intermediate inputs. Increasing marginal costs implied by t_i, the shift parameter associated with the stepped supply function, being greater than zero, would indicate that at some stage domestic marginal costs will exceed the price of imported intermediate inputs and at this point it would become profitable to use imported intermediate inputs. The introduction of increasing costs could reduce the possibility of knife-edge behavior in the intermediate input sector of the model. It should be emphasized that the above situation is a theoretical possibility and whether it will occur or not will depend on the particular economy being studied.

Condition (vi) states that exports should be undertaken till the point where domestic price equals marginal revenue from foreign sales. For traditional exports, the downward sloping export demand curves ensure that there are limits to export expansion. The small country assumption is made for the other sectors.

The interpretation of the first order conditions indicate that both from the production and consumption sides, the knife-edge properties of the traditional general equilibrium trade model are likely to be reduced. The extent to which these theoretical possibilities effectively solve the specialization problem can only be gauged by an empirical implementation of the model.

IV. EMPIRICAL IMPLEMENTATION OF THE MODEL

The model[2] has been empirically implemented with a twelve sector classification. The traditional export sectors are represented by the cash crops, food and beverages, and textiles sectors. Machinery, metal and steel, light industry, and chemical and petroleum constitute the import-substituting sectors. The major agricultural sectors include food cash crops and foodgrains. Domestic industry and services are the non-traded goods sector. The sectoral classification and base year were determined by the 1964–65 input-output table which is currently available. International trade restrictions are introduced through quotas which have primarily been used to control imports. The nonlinear constrained optimizing problem was solved by using Fiacco and McCormick's sequential unconstrained minimization technique [10].

The presentation of the results will focus on three issues. First, the model is run with base year quantitative restrictions to evaluate how well it replicates the historical solution. Second, the free trade solution and its divergence from the trade distorted situation are analyzed. Finally, the behavior of the dual variables in the optimal solution are described.

The mathematical model described in section II can be viewed as one that generates a competitive general equilibrium solution including prices from the dual. Since no economy meets all of the assumptions of perfect competition, the model's optimal solution will not be identical to the actual historical solution. Differences between the model's optimal solution and the economy's actual solution could be explained both by institutional features not included and the modelling assumptions made. The most important institutional features not included in the model are product and factor market distortions and direct government intervention in the domestic sector of the economy. In addition, the assumption of capital mobility implies long-run equilibrium. The historical solution may refer to short-run equilibrium, in which case the model solution is strictly not comparable to the historical solution.

To keep any model tractable, some simplifying assumptions have to be made. The question that has to be answered is not whether the economy meets these assumptions, but rather how closely it meets them. This can be answered by comparing the solution given by the model to the historically given base year values. Table I compares the protected optimal with base year values. It indicates that our formulation of the model in the trade distorted situation gives a good description of the economy in the base year. While the assumptions made in regard to quantitative restrictions on imports and the production functions con-

[2] A detailed description of the sources of data and the methodology used in fitting Indian data to the model is contained in [1] [2].

TABLE I

COMPARISON OF PROTECTED OPTIMAL AND BASE YEAR VALUES

Sector	(1) $C_{id}+C_{im}$	(2) X_i	(3) $C_{im}+M_i$	(4) E_i	(5) K_i	(6) L_i
1. Food cash crops	108	105	100	—	110	111
2. Cash crops	94	102	—	70	105	102
3. Foodgrains	102	103	100	—	106	106
4. Machinery*	104	97	100	—	97	95
5. Metal & steel*	111	100	100	—	101	98
6. Mining	117	91	100	—	88	86
7. Light industry*	110	88	100	—	73	71
8. Food & beverages	100	98	100	100	100	97
9. Textiles	110	106	100	100	108	105
10. Chemicals & petroleum*	102	97	100	—	92	90
11. Domestic industry	109	100	—	—	101	98
12. Services	111	89	—	—	88	90

* Labor is expressed in thousands of workers and the rest in millions of rupees.

TABLE II

SHADOW PRICES ASSOCIATED WITH PROTECTED SOLUTION

Sector	(1) π_i	(2) π^*_i	(3) π_{vi}	(4) π_{qi}	(5) π_{ti}
1. Food cash crops	0.09823	0.09823	0.07621	0.04069	0.03015
2. Cash crops	0.08481	0.08481	0.07075	—	0.01819
3. Foodgrains	0.08784	0.08784	0.07458	0.02057	0.03104
4. Machinery*	0.14060	0.14060	0.06957	0.10857	
5. Metal & steel*	0.14465	0.14465	0.06996	0.11298	
6. Mining	0.14575	0.14575	0.06687	0.06641	π_K=0.01522
7. Light industry*	0.20023	0.20023	0.05765	0.16000	π_L=0.02734
8. Food & beverages	0.14715	0.14715	0.07031	0.06226	π_f=0.08491
9. Textiles	0.14514	0.14514	0.07031	0.06026	
10. Chemicals & petroleum*	0.25580	0.25580	0.06585	0.21335	
11. Domestic industry	0.26819	0.26819	0.06989	—	
12. Services	0.10326	0.10326	0.07015	—	

* See Table I.

tributed to this results, the proximity of the trade distorted solution to the actual historical solution suggests that our modelling assumptions enable us to use the optimal trade distorted solution as the starting point for the comparative static exercises.

Table III gives the numerical values of the primal variables in the trade distorted situation while Table II gives the duals or shadow prices associated with each of the constraints. The import-substituting sectors are starred. Table III indicates that foodgrains and food cash crops, the major consumer goods producing sectors are the largest. Textiles, domestic industry, and food and beverages are next in

TABLE III

PROTECTED SOLUTION

Sector	(1) C_{id}	(2) C_{im}	(3) X_i	(4) I_i	(5) M_i	(6) E_i	(7) K_i	(8) L_i
1. Food cash crops	21,791.6	204.0	39,472.4	17,680.8	—	—	31,613.8	33,002.6
2. Cash crops	938.3	—	4,870.2	2,838.8	—	1,093.0	7,186.4	5,814.5
3. Foodgrains	36,479.6	1,738.8	49,626.2	13,146.6	46.2	—	36,466.2	39,252.6
4. Machinery*	982.1	247.1	9,368.9	8,386.9	3,713.9	—	24,448.3	10,226.8
5. Metal & steel*	2,015.0	30.7	9,349.3	7,334.4	1,307.3	—	24,810.6	10,106.1
6. Mining	93.8	—	2,062.4	1,968.6	200.0	—	4,852.9	2,341.8
7. Light industry*	2,090.4	45.7	6,316.8	4,226.5	787.3	—	10,018.4	7,740.8
8. Food & beverages	17,760.0	270.5	21,332.8	3,178.0	175.5	395.0	39,096.6	33,090.2
9. Textiles	12,016.9	121.4	22,943.5	8,035.6	14.6	2,891.0	41,846.3	35,700.0
10. Chemicals & petroleum*	2,207.9	523.6	5,851.5	3,643.5	796.4	—	18,528.3	3,774.9
11. Domestic industry	1,798.8	—	21,895.8	20,097.1	—	—	70,058.2	16,956.6
12. Services	3,178.7	—	9,213.9	6,035.2	—	—	20,668.0	12,131.2

* See Table I.

TABLE IV

PERCENTAGE DEVIATION OF FREE TRADE FROM PROTECTED SOLUTION OF PRIMAL VARIABLES

Sector	(1) C_{id}	(2) C_{im}	(3) X_i	(4) I_i	(5) M_i	(6) E_i	(7) K_i	(8) L_i
1. Food cash crops	3.1	−3.0	5.4	8.3	—	—	20.8	7.8
2. Cash crops	13.1	—	13.2	4.4	—	36.1	25.4	12.0
3. Foodgrains	−8.2	−22.0	18.1	9.8	−100.0	Zero to 10,685.7	57.3	40.4
4. Machinery*	74.7	162.2	−81.7	−100.0	202.6	—	−83.4	−85.2
5. Metal & steel*	64.7	172.6	−64.5	−100.0	385.2	—	−66.8	−70.4
6. Mining	22.9	—	−12.3	−14.0	−100.0	—	−9.6	−19.3
7. Light industry*	108.6	197.4	−31.0	−100.0	399.7	—	−52.0	−57.2
8. Food & beverages	8.1	3.5	7.9	6.4	−100.0	12.7	15.6	3.2
9. Textiles	15.5	2.1	15.3	13.3	−100.0	20.5	23.5	10.3
10. Chemicals & petroleum*	98.2	260.0	−25.2	−100.0	430.0	—	−49.7	−55.1
11. Domestic industry	23.0	—	1.1	−0.9	—	—	6.8	−4.7
12. Services	28.0	—	3.0	−10.2	—	—	9.2	−2.5

* See Table I.

order of importance. Except for foodgrains and chemicals and petroleum, the shares of imported consumption goods in total imports of a sector are small.

The dual prices are useful in analyzing the behavior of I_i and M_i. The commodity balance constraints indicate that imported and domestically produced intermediate inputs are perfect substitutes. A high π_i implies that domestic cost of producing the good i is relatively large. Table II shows that of the traded goods chemicals and petroleum. Light industry, mining, metal and steel, and machinery have high costs. Given the assumption of perfect substitutes, foreign exchange will first be allocated to the sector with the highest resource cost, and only when the import quota or ceiling is reached, allocation of foreign exchange will switch to the next highest cost sector. In this way, foreign exchange is allocated in a hierarchical manner. The shadow prices π_{qi} associated with the import quotas indicate that in all nine sectors they are binding. An interesting result is that for food and beverages and textiles, the same commodity is being exported and imported since both E_i and M_i are greater than zero. This is explained by the downward sloping export demand functions for the traditional sectors, the infinite elasticity of substitution between E_i and M_i and the hierarchical nature of foreign exchange allocation.

Free Trade Solution

Tables IV and V give the percentage deviation of the primal and dual variables of the free trade solution from the protected solution. Though no sector disappears in a move to free trade, there is complete substitution of I_i by M_i in the four import-substituting sectors. Thus, we see that the stepped supply function does not alleviate the knife-edge behavior in the intermediate input part of the model. The values of the dual variables associated with the intermediate input and

TABLE V

PERCENTAGE DEVIATION OF SHADOW PRICES OF FREE TRADE FROM THE PROTECTED SOLUTION

	Sector	(1) π_i	(2) π^*_i	(3) π_{vi}	(4) π_{ti}
1.	Food cash crops	9.8	9.8	10.5	16.6
2.	Cash crops	0.1	0.1	6.9	21.0
3.	Foodgrains	23.4	23.4	28.5	51.7
4.	Machinery*	−35.2	−56.8	−12.5	
5.	Metal & steel*	−31.2	−58.5	−9.8	$\pi_K = -3.5$
6.	Mining	−7.9	−7.9	−0.5	$\pi_L = 8.1$
7.	Light industry*	−45.7	−61.9	−32.9	$\pi_f = 89.0$
8.	Food & beverages	4.7	4.7	3.3	$U = 13.7$
9.	Textiles	−1.9	−1.9	3.3	
10.	Chemicals & petroleum*	−42.5	−68.5	−35.2	
11.	Domestic industry	−8.0	−8.0	1.9	
12.	Services	−11.6	−11.6	2.3	

* See Table I.

exogenous demand constraints, π^*_i, are less than the shadow price, π_i, which correspond to the commodity balance constraints in the four import-substituting sectors. This condition is required for complete substitution of I_i by M_i.

The theoretical possibility of the simultaneous use of domestic and imported inputs in the import-substituting sectors not being borne out in the actual free trade solution can be explained by comparing marginal domestic cost with imported price. I_i is replaced by M_i despite the fact that as output declines in machinery, metal and steel, light industry, and chemicals and petroleum, efficiency increases owing to the nature of the stepped supply function. There are two opposing forces at work. First, there is a cost decline resulting from an increase in efficiency as output decreases. Second, the removal of the quotas reduces the cost of using imported items in the economy. The second effect outweighs the first in the import-substituting sectors and I_i is replaced by M_i. Interestingly, in foodgrains, mining, food and beverages, and textiles, M_i is completely replaced by I_i. Here the domestic cost of production is lower than the imported cost leading to complete substitution.

A comparison of columns (1) and (2) in Tables IV and V indicates an overall inverse relationship between consumption and dual prices for both domestically produced and imported goods. There is a one to one relationship between value added π_{v_i} and gross output for all the sectors. Through value added and output change in the same direction, factor rental changes become important in resource reallocation. A modest, 4.6 per cent, increase in the wage rental ratio causes substitution of capital for labor in the domestic industry and services sector. Here output and capital employed increase but labor employed decreases.

The largest increases in output occur in foodgrains, textiles, cash crops, and food cash crops. There is substantial resource reallocation in favor of these sectors implying that comparative advantage for India lies in these sectors. The outputs of all four import-substituting sectors decline significantly. Given the nature of the commodity balance constraints, it is not surprising that output decline is accompanied by increases in C_{id} in these sectors. A surprising result is the increase in output of foodgrains together with a decline in foodgrains consumption. This is explained by large foodgrain exports which imply that domestic price is determined by world price multiplied by the shadow exchange rate. The shadow exchange rate increases sharply.

The move to free trade results in a 13.2 per cent increase in welfare. The volume of trade increases appreciably. The relatively small increases in traditional exports are explained by world demand constraints. The substantial exports of foodgrains are caused by world demands and no lower bounds being put on domestic consumption. Large increases in imports of machinery, metal and steel, light industry, and chemicals and petroleum occur despite a large 89 per cent devaluation and the assumption of the stepped supply function for these sectors.

V. CONCLUSION

The objective of this paper has been to develop and test a model for understanding and planning the sectoral allocation of resources for an open economy. The problem of specialization inherent in any open economy model is sought to be tackled through introducing nonlinearities from the supply as well as demand sides. Factor mobility ensures the long-run nature of the model.

The empirical implementation of the model suggests that it tracks the economy sufficiently closely to provide a benchmark for the comparative static exercise. In the free trade solution, the specialization problem is effectively tackled. This is done primarily through substitution between domestically produced and imported consumption goods. For intermediate inputs, the assumption of perfect substitutability between domestically produced and imported goods, leads to knife-edge behavior with either domestically produced or imported intermediate inputs being solely used. However, certain intermediate inputs which were being imported in the protected situation are domestically produced in the free trade solution. This shows that though the knife-edge behavior persists it is not biased in one direction only. The correspondence between changes in value added and output is clearly seen once the general equilibrium repercussions of a move to free trade are taken into account. The sensitivity of resource reallocation to changes in factor rentals is highlighted.

In the Indian context, the overwhelming conclusion is that comparative advantage lies in agriculture and related sectors. There is a large movement of resources from the import substituting to the agricultural sectors in the free trade solution with the percentage movement of capital being larger than the percentage movement of labor. Capital intensity in the agricultural sectors increases. Foodgrains become the most important export item. But the subsequent increase in foodgrain prices triggered largely by a 89 per cent devaluation of the shadow exchange rate leads to a decline in foodgrains consumption. In a poor country, this could aggravate the poverty problem. Thus, the increase in efficiency resulting from a move to free trade will have to be weighed against the loss on equity considerations.

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