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FORMATION OF EXPECTATIONS AND LEARNING IN THE MARKET

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I. INTRODUCTION

RECENT decades have seen intensive theoretical and empirical investigation of the way that economic agents form their expectations of future events. In making these investigations, many economists find the hypothesis of rational expectations attractive for application to a wide range of phenomena. In particular, they have combined the hypothesis of rational expectations with the new-classical macroeconomic theory, to provide radical policy implications that demonstrate the ineffectiveness of anticipatory monetary policy.

Although the literature includes many studies on the hypothesis of rational expectations, few are devoted to shedding light on the hypothesis's associations with the learning process.[1] The first step for agents in the process of learning is to acquire information about the economic structure. That information is then used to make more precise inferences about the structure. Since economies are in constant flux, the information gathering and inference creation process is continuous and information on a particular impact usually becomes available only after a lag in time. Only over the long run can agents find out what the true structure of a stationary economy is and only then can subjective and rational expectations merge. During transitional periods, however, subjective expectations still differ from objective, mathematical expectations and the degree of that difference is related to the degree of learning.

The hypothesis of catastrophic expectations is a more general expectations hypothesis that makes the learning process an explicit part of expectation formation. It is a hypothesis that embodies both rational and nonrational expectations. Insufficient learning levels prevent the expectations formed from coinciding with rational expectations, but as agents acquire and process information, the formed expectations begin to approximate rational expectations. There is also a likelihood with the hypothesis of catastrophic expectations of sudden jumps in expectations, a phenomenon seen quite frequently in volatile markets such as stocks, commodities, and foreign exchange. The empirical examination of the hypothesis here uses

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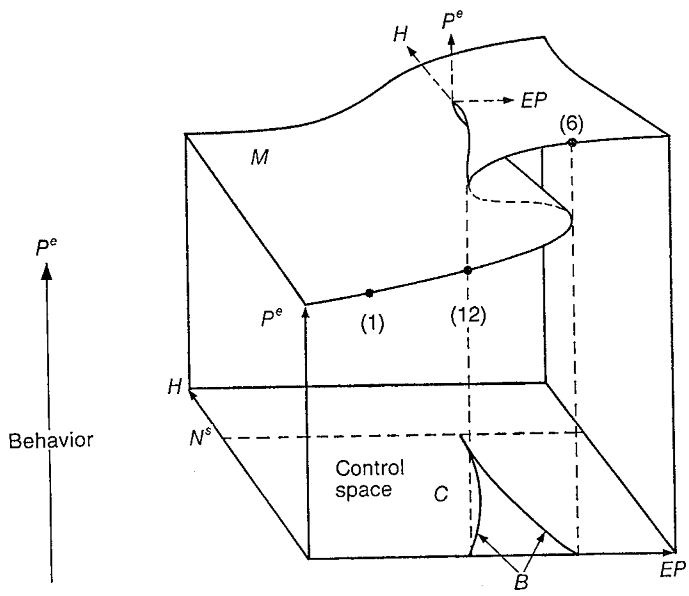
[1] See [3] [5] [4] on learning in the process of forming expectations.

Fig. 1. The Cusp Catastrophe Graph: The Dynamics of Price Expectations



time-series data on the price of copper futures. The evidence obtained supports the expectations hypothesis and indicates the presence of a learning process in the market for copper on the London Metals Exchange, a fact that market studies should not ignore.

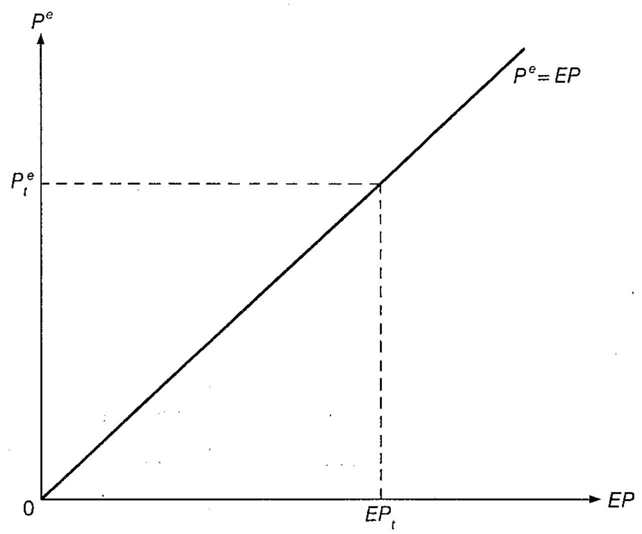
In Section II, the hypothesis of catastrophic expectations is formulated mathematically. Section III discusses the construction of a model that takes into account divergent expectations of agents resulting from differences between them in learning level. This is the model used in Sections IV and V to estimate the degree of learning in the copper market. Section VI suggests some policy implications and gives the conclusions reached by the authors. It also shows that price stabilization temporarily and cyclically improves aggregate welfare and makes timing of policy implementation critical.

II. FORMULATING A HYPOTHESIS OF CATASTROPHIC EXPECTATIONS

The hypothesis of catastrophic expectations incorporates nonrational and rational expectations.

Thom's [14] discoveries in catastrophe theory have been applied to many fields. Kuchiki [11] used them to make models of expectations in economics and obtain the "hypothesis of catastrophic expectations." The hypothesis of catastrophic

Fig. 2. Rational Expectations ($H \geq N^s$)



expectations was derived by slightly modifying the system of Hull [9], a neo-behavioral psychologist who examined the development of learning in human behavior.

Here, E is the potential reaction function of price expectations. Crucial to catastrophe theory is the kind of control variables chosen, and the Hull [9] system's virtue is its ability to use predictions of relevant economic theory (EP) and levels of learning (H) as control variables. Catastrophe theory has proved that in elementary catastrophes, "cusp" catastrophes which we apply in this paper are relevant to systems where behavior depends on two control factors.

Visualizing a control space C in Figure 1 as a horizontal plane with coordinates EP and H gives,

$$c = (EP, H), \quad (c \in C).$$

The hypothesis of catastrophic expectations is a cusp catastrophe in which the predictions of relevant economic theory EP are "normal factors" and level of learning H is a "splitting factor."[2] These terms are used because:

Case 1: If H is high ($H \geq N^s$), then the reaction potential function of P^e will be unimodal, and that will call for greater P^e. In this case, the hypothesis of

[2] People first compute predicted values that are predicated on the available information at time t, which is written as $E_t(P_{t+1})$. However, there is a time lag between the occurrence of an economic event and the acquisition of data about it. When oil crisis or other sharp economic fluctuations, or overall political change causes permanent structural changes in an economy, people do not use predicted values for their expectations $P_{t+1,t}{}^e$. They use predicted values EP to determine the expected value P^e as determined by their level of learning H and the predicted value EP.

Fig. 3. Unimodal Distribution of Price Expectations

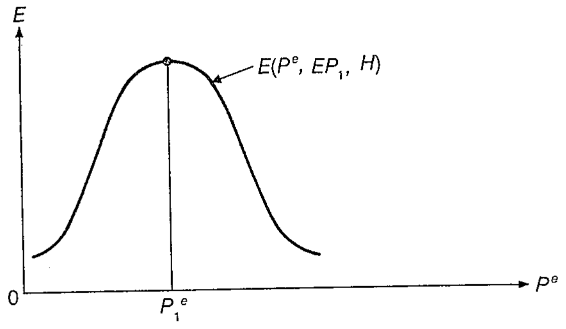
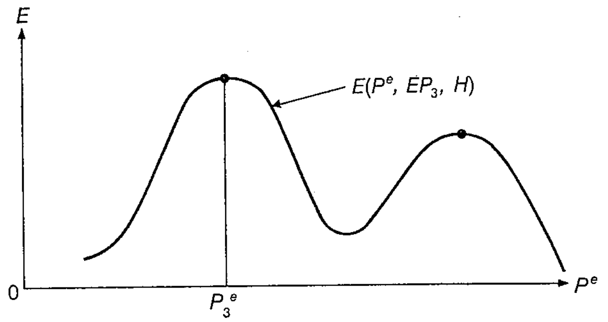


Fig. 4. Bimodal Distribution of Price Expectations



rational expectations holds, that is to say, $P^e = EP$ in Figure 2. The reaction potential function of P^e is shown in Figure 3.

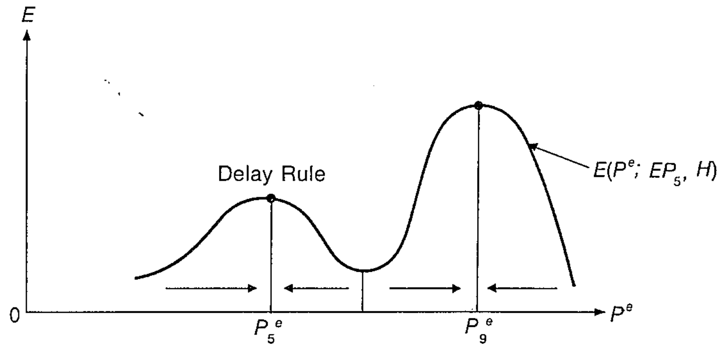
Case 2: If H is low $(H < N^s)$, and the prediction moderate, then the unimodal distribution in Figure 3 will be split bimodally as in Figure 4. Here N^s is the critical level of learning.

Case 3: If H is low $(H < N^s)$, but the prediction is very great or small, then the distribution will be unimodal. The reaction potential function is again depicted in Figure 3.

Behavior variable P^e, a vertical coordinate in a behavior space (see Figure 1), is defined for the model.

Under the Delay Rule, expectation P^e changes in the same direction in which the reaction potential E locally increases. When minimum expectations divide the behavioral variable's coordinates into several parts, the expectation moves to the maximum point that dominates the section where the current price is. When

Fig. 5. Delay Rule



this is applied to the distribution in Figure 5, the directions of the arrows indicate the directions of change in expectations according to the Delay Rule. The graph's slope determines the arrow directions at each point, because it indicates which direction causes an increase in the reaction potential. The arrows point toward the local maxima, and therefore, under the Delay Rule, the expectation P^e changes until it is at one of the local maxima.[3]

The Delay Rule can be related to Hull's concept of "reaction latency." The reasons for choosing this rule are: (1) there is strong likelihood that the public's biased information prevents their looking at the aggregate economy from a global perspective; (2) it is easier to determine the global potential function E than it is to determine the local direction in which moves are made toward greater density, the latter determination being one that is often made "intuitively;" and (3) drastic changes in expectations take longer.

Figures 2 and 6C show the difference between hypothesis of catastrophic expectations and hypothesis of rational expectations. Figure 2 shows a hypothesis of rational expectations that equates public expectations with predictions by a relevant economic model. Figure 6C shows that the hypothesis of catastrophic expectations also implies the public's expectations deviate from what relevant economic theory predicts, and that deviation is described by

$$P_{t,t-1}{}^e = U_{t-1}E_{t-1}(P_t), \qquad (1)$$

where

P_t = actual price level prevailing in period t,

$P_{t,t-1}{}^e$ = agent's expectation at the end of period $t-1$ for t's price level, and

$E_{t-1}(P_t)$ = mathematical expectation of variable P_t conditioned on the information at the end of period $t-1$.

Two different phases are characteristic of the hypothesis of catastrophic expectations:

[3] See [16].

Fig. 6.

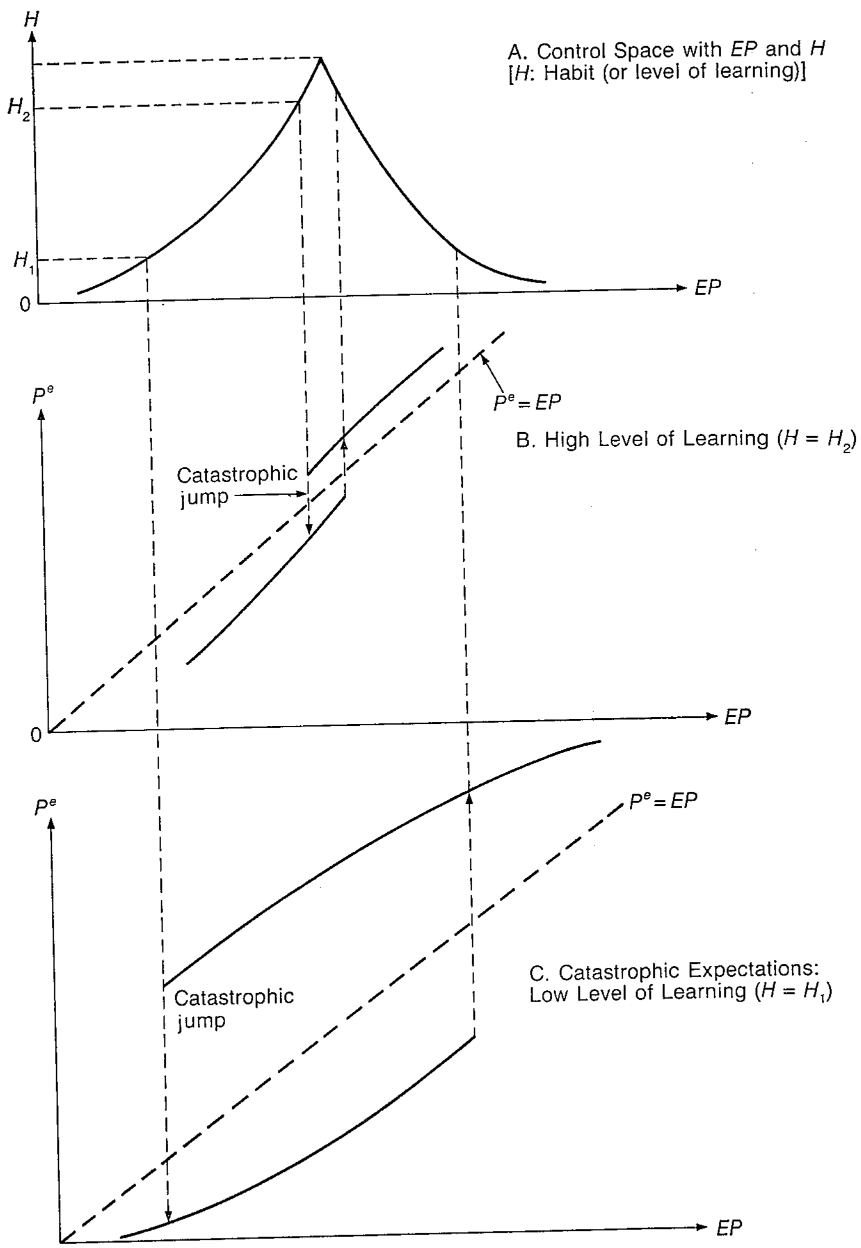
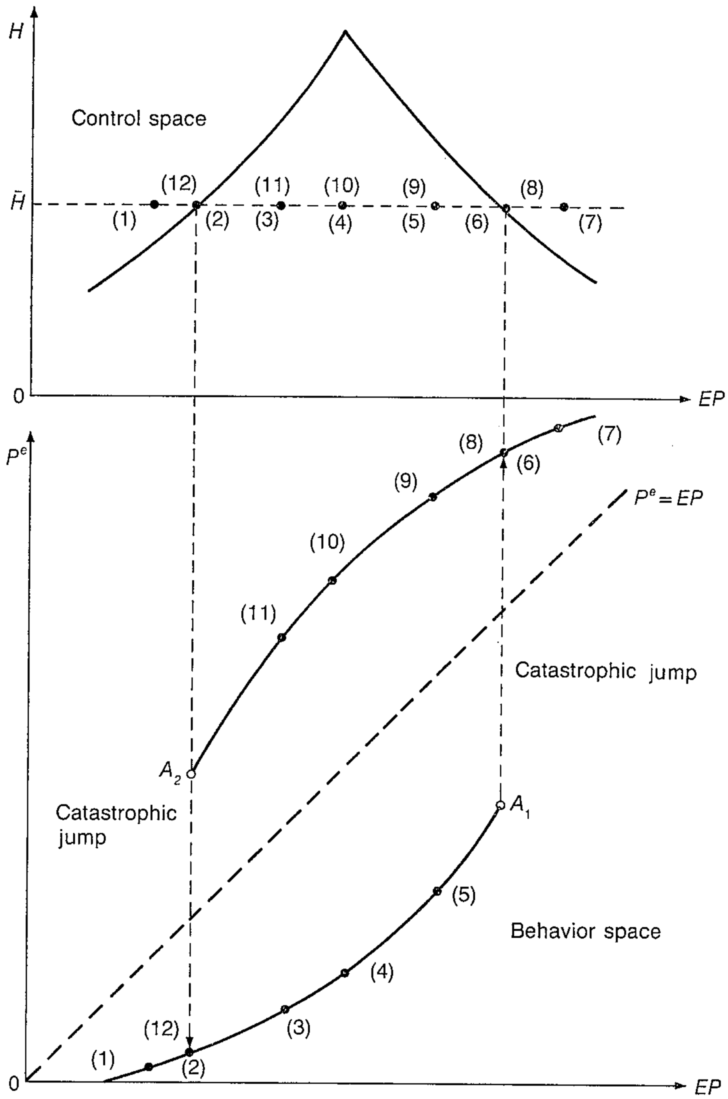

H
H_2
H_1
0
EP
A. Control Space with EP and H
[H: Habit (or level of learning)]
P^e
$P^e = EP$
B. High Level of Learning ($H = H_2$)
Catastrophic
jump
0
EP
P^e
$P^e = EP$
C. Catastrophic Expectations:
Low Level of Learning ($H = H_1$)
Catastrophic
jump
EP

Fig. 7. Dynamics of Price Expectations: Control Space and Behavior Space

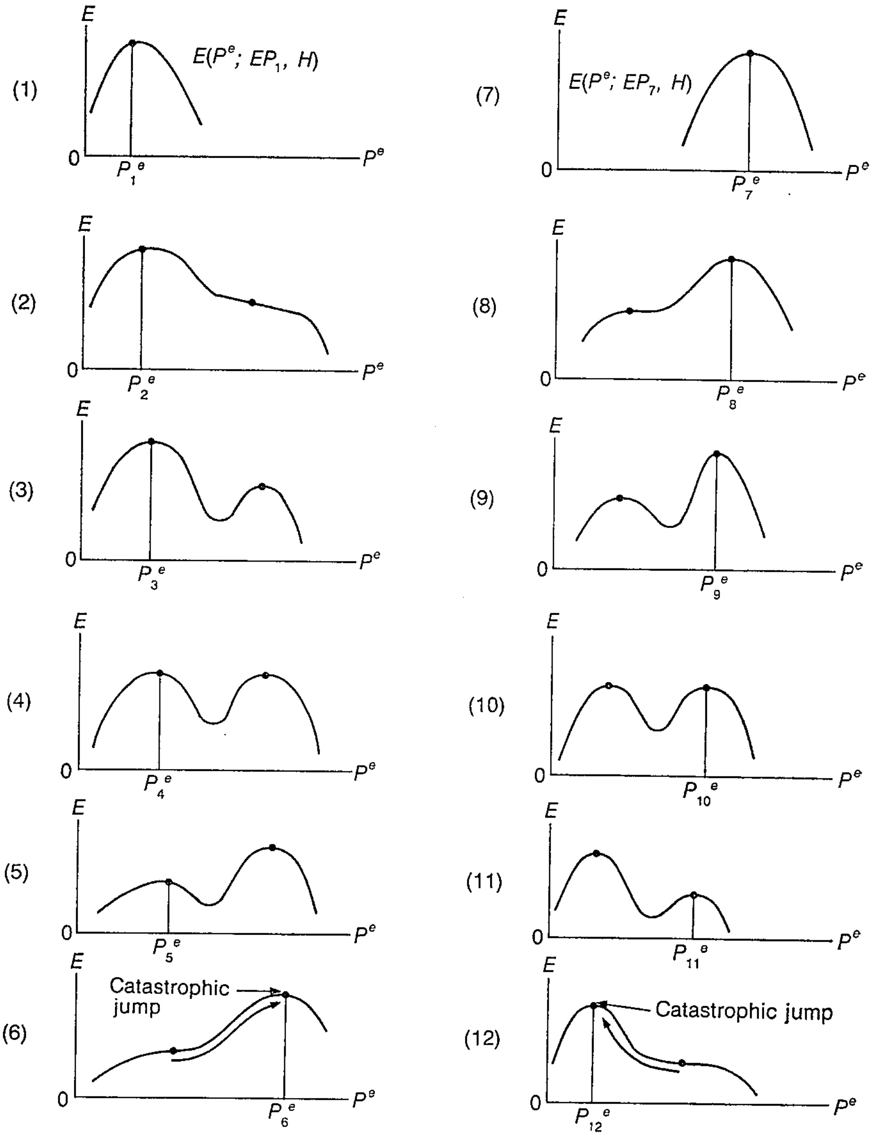


(a) Phase less-than-one U_{t-1} (<1).

(b) Phase more-than-one U_{t-1} (>1).

Suppose that relevant economic theory (*EP*) predicts gradual increase while learning level (*H*) remains steady, but at low levels. Point (*EP*, P^e) in Figure 7, then shifts along the lower curves through points (1), (2), (3), (4), and (5). At point A_1, public expectations of value undergo a change, what is called a

Fig. 8. Dynamics of Price Expectations: Potential Functions



catastrophic jump. The same analogy holds for Figure 7 where the shift starts at points (7) to (11) along the upper curve and the catastrophic jump is at point A_2. Figure 8 shows potential functions that correspond to points (1)–(12) in Figure 7. Figure 6 depicts dynamic processes in which the distance of catastrophic jump decreases as level of learning H rises. Figure 6C shows a low-level learning stage, while Figure 6B shows a stage where learning level H has risen. U_{t-1}'s values ultimately converge at unity as the level of learning exceeds critical point N^s. The ultimate situation occurs when the hypothesis of rational expectations has a U_{t-1} magnitude that always equals unity. That is,

$$P_{t,t-1}{}^e = E_{t-1}(P_t).$$

These descriptive explanations may now be rigorously formulated. According to the catastrophe theory, the reaction potential function has the form:[4]

$$E = E_c(P^e) = -(P^e - k)^4/27(k-j)^2 - \min(0, H - N^s)(P^e - k)^2/2 + (EP - j)(P^e - k) + m, \quad (2)$$

where

N^s = critical level of learning, i.e., rational expectations emerge if $H \geq N^s$, and nonrational expectations emerge if $H < N^s$,

k, j, m = parameters.

This equation should hold for the manifold M in Figure 1:

$$(\partial E_c)/(\partial P^e) = 0$$

or

$$EP = [4(P^e - k)^3]/[27(k-j)^2] + (H - N^s)(P^e - k) + j, \quad (3)$$

then

$$(\partial EP)/(\partial P^e) = (4/9)(P^e - k)^2/(k-j)^2 + (H - N^s). \quad (4)$$

An $H \geq N^s$ simplifies equation (3) to

$$EP = (4/27)(P^e - k)^3/(k-j)^2 + j. \quad (5)$$

A Taylor expansion of equation (5) around $k + (3/2)(j-k)$, which ignores the second or higher order terms, yields,

$$EP \fallingdotseq P^e. \quad (6)$$

Equation (6) implies an almost rational formation of expectations. It should also be noted that when $H \geq N^s$,

$$(\partial EP)/(\partial P^e) > 0,$$

that is, no catastrophic jump ever occurs. Therefore, it is clear that when $H \geq N^s$, the hypothesis of rational expectations holds, a situation that corresponds to case 1.

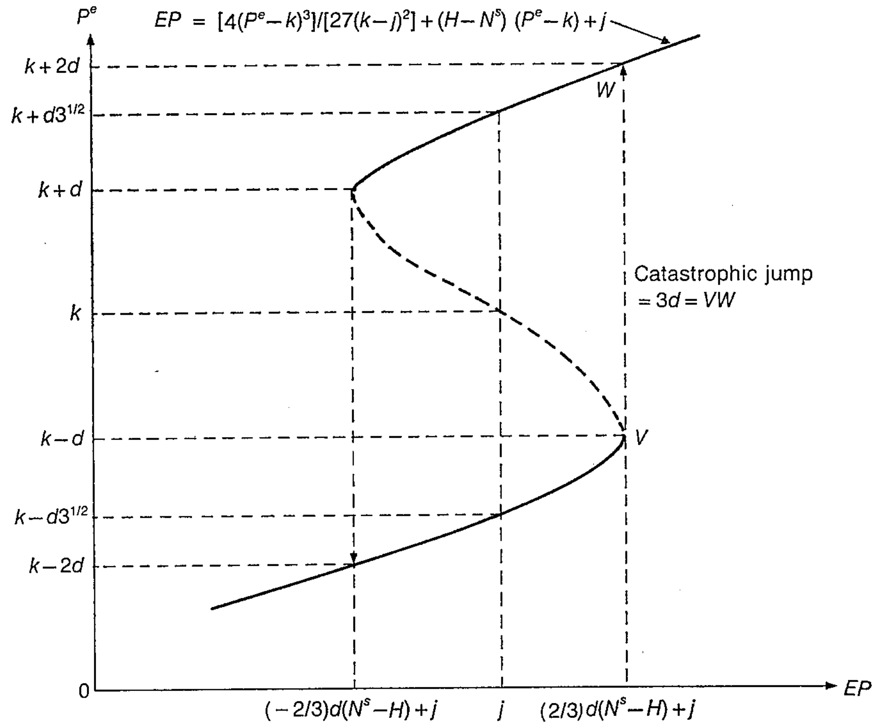
In Figure 9, $H < N^s$. Here, the distance of the catastrophic jump, VW, is $3d$ where $d = (3/2)|k - j|(N^s - H)^{1/2}$.

[4] The standard unholding is

$$E = E_c(x) = \pm x^4 + ax^2 + bx,$$

where x is a behavior variable, a is a splitting factor, and b is a normal factor. In the model, P^e is equivalent to a behavior variable, H to a splitting factor, and EP to a normal factor. Note that in equation (2), k, j, and m are shift parameters.

Fig. 9. An Example of Catastrophic Jumps



Note that $(N^s - H)$ measures insufficient learning. It implies that the distance of catastrophic jump decreases as learning approaches critical level N^s. Therefore, if the value predicted by economic theory is quite small or quite large, which is equivalent to

$$EP < j - [(2/3)d(N^s - H)],$$

or

$$EP > j + [(2/3)d(N^s - H)],$$

and

$$H < N^s,$$

then it is obvious from Figure 9 that there will be no catastrophic jump. This corresponds to case 3. However, if $H < N^s$ and the prediction from the economic theory takes moderate values, there are possibilities that catastrophic jumps occur, i.e., case 2. In cases 2 and 3, P^e is not equal to EP, and expectations are formed nonrationally.

This rigorous demonstration shows the emergence of a different phase of expectation formation that is dependent on the level of learning and the magnitude of the value predicted by economic theory.

The hypothesis of catastrophic expectations thus turns out to be quite general, because it embodies hypotheses of both rational and nonrational expectation. When the level of learning is sufficiently high, agents form expectations that prove rational, but when the level of learning is low, insufficient information leads to nonrational expectations. Furthermore, a low level of knowledge brings ambiguity to the formation of expectations when the values that economic theory predicts are neither large nor small, and those low levels may cause sudden jumps in expectations.

III. FUTURES MARKETS AND DEGREE OF LEARNING

U_t is the key variable for expressing degree of learning in the market. What is required now is the construction of a model of the futures market and an explanation of how the degree of learning relates to the equilibrium futures price.

The model constructed here is basically taken from [12, Chap. 13]. Let P_t^s be the spot price for copper at time t, which is unknown three months before at time $t-3$. People contract to sell or buy copper at time $t-3$ for time t, at a value per contract of $P_{t,t-3}{}^f$. Income for representative firm i at time t is:

$$Y_t^i = P_t^s(q_i - Z_i) + P_{t,t-3}{}^f Z_i - C_{t-3}Z_i - b_{t-3}X_i,$$

where

X_i = quantity of input,
b_{t-3} = input price,
q_i = amount of output produced, related to X_i by the production function $q_i = q_i(X_i)$,
Z_i = number of unit futures contracts supplied by firm i, and
C_{t-3} = transaction costs for futures contracts. This will be explained later.

The following exponential utility function is assumed:

$$W_i(Y_t^i) = -\exp(-A_{1i}Y_t^i),$$

where

A_{1i} = Arrow-Pratt measures of absolute risk aversion.

The ith firm chooses Z_i to maximize expected utility, which, if Y_t^i is normally distributed and conditional on $P_{t,t-3}{}^f$, is equivalent to maximizing,

$$E_{t-3}(Y_t^i) - (1/2)A_{1i}Var_{t-3}(Y_t^i). \qquad (7)$$

An optimal Z_i satisfies:

$$Z_{1i} = [P_{t,t-1}{}^f - C_{t-3} - E_{t-3}(P_t^s)] / [A_{1i}Var_{t-3}(P_t^s)]. \qquad (8)$$

Taking into account the degree of divergence of subjective expectations and of variance from the mathematical conditional expectation and variance, which is caused by an insufficient level of learning, equation (8) is rewritten as[5]

$$Z_{1i} = [P_{t,t-3}{}^f - C_{t-3} - U_{t-3}{}^i E_{t-3}(P_t{}^s)] / [A_{1i} V_{t-3}{}^i Var_{t-3}(P_t{}^s)] + q_i. \qquad (9)$$

Similarly a pure speculator j maximizes

$$W_j(Y_t{}^j) = -\exp(-A_{2j} Y_t{}^j),$$

where

$$Y_t{}^j \equiv P_t{}^s(-Z_{2j}) + P_{t,t-3}{}^f Z_{2j} - C_{t-3} Z_{2j}.$$

Or under the same assumptions as above,

$$E_{t-3}(Y_t{}^i) - (1/2) A_{2j} Var_{t-3}(Y_t{}^i).$$

The first order condition yields the following optimal forward contracts

$$Z_{2j} = [P_{t,t-3}{}^f - C_{t-3} - E_{t-3}(P_t{}^s)] / [A_{2j} Var_{t-3}(P_t{}^s)]. \qquad (10)$$

If the divergence in expectations and the variances are taken into consideration, equation (10) can be rewritten as

$$Z_{2j} = [P_{t,t-3}{}^f - C_{t-3} - U_{t-3}{}^j E_{t-3}(P_t{}^s)] / [A_{2j} V_{t-3}{}^j Var_{t-3}(P_t{}^s)]. \qquad (11)$$

For the futures market to be in equilibrium, net sales must be zero, or added over firms and speculators:

$$0 = \sum_i Z_{1i} + \sum_j Z_{2j},$$

or

$$E_{t-3}(P_t{}^s) = (\theta_{t-3}{}^1/\theta_{t-3}{}^2) P_{t,t-3}{}^f - (\theta_{t-3}{}^1/\theta_{t-3}{}^2) C_{t-3} + (1/\theta_{t-3}{}^2) Q_{t-3} Var_{t-3}(P_t{}^s), \qquad (12)$$

where

$$\theta_{t-3}{}^1 \equiv \sum_i (1/A_{1i} V_{t-3}{}^i) + \sum_j (1/A_{2j} V_{t-3}{}^j),$$

$$\theta_{t-3}{}^2 \equiv \sum_i (U_{t-3}{}^i/A_{1i} V_{t-3}{}^i) + \sum_j (U_{t-3}{}^j/A_{2j} V_{t-3}{}^j),$$

$$Q_{t-3} \equiv \sum_i q_i.$$

Equation (12) is the equilibrium between the expected spot price and the futures price. Note that complete learning $V_{t-3}{}^i = V_{t-3}{}^j = U_{t-3}{}^i = U_{t-3}{}^j = 1$ reduces equation (12) to

$$E_{t-3}(P_t{}^s) = P_{t,t-3}{}^f + (1/A) Q_{t-3} Var_{t-3}(P_t{}^s) - C_{t-3},$$

5 $$Y_t{}^i = P_t{}^e(i)(q_i - Z_i) + P_{t,t-3}{}^f Z_i - C_{t-3} Z_i - b_{t-3} x_i,$$

where $P_t{}^e(i)$ is the price forecast by firm i. Suppose that

$$P_t{}^e(i) = E_{t-3}(U_{t-3}{}^i P_t{}^s).$$

Then allowing $V_{t-3}{}^i$ to equal $(U_{t-3}{}^i)^{-2}$ gives equation (9).

where

$$A \equiv \sum_i (1/A_{1i}) + \sum_i (1/A_{2j}).$$

The coefficient of $P_{t,t-3}{}^f$, $(\theta_{t-3}{}^1/\theta_{t-3}{}^2)$ can be interpreted as the reciprocal of the weighted average of individual learning levels ($U_{t-3}{}^i$) in the market, with the weights being

$$(1/A_{1i}V_{t-3}{}^i)/[\sum_i (1/A_{1i}V_{t-3}{}^i) + \sum_j (1/A_{2j}V_{t-3}{}^j)].$$

The expression $(\theta_{t-3}{}^1/\theta_{t-3}{}^2)$ is then rewritten as $1/U_{t-3}$, with U_{t-3} as the market aggregated degree of learning. Equilibrium in the futures market for commodities enables the derivation of an equation that associates expected spot prices with futures prices. That allows a variety of transaction costs in the following form:

$$E_{t-3}(P_t{}^s) = a_{t-3} + (1/U_{t-3})P_{t,t-3}{}^f, \qquad (13)$$

where

$a_{t-3} = -(1/U_{t-3})(S_{t-3} + r_{t,t-3}P_{t-3}{}^M) + (1/\theta_{t-3}{}^2)Q_{t-3}Var_{t-3}(P_t{}^s)$,

$P_{t,t-3}{}^f$ = value of contract made at period $t-3$ for delivering commodities in three months,

$P_t{}^s$ = spot price,

$P_{t-3}{}^M$ = minimum margin per futures contract,

S_{t-3} = transaction cost per futures contract,

$r_{t,t-3}$ = three-month interest rate prevailing at period $t-3$,

Q_{t-3} = output level at period $t-3$.

As an example of transaction costs, a_{t-3}, the conditions on the American Board of Trade (New York) in January 12, 1983 may be considered. The costs to be paid to brokers are (1) commission fee per contract: $S=$ U.S.\$62.50, and (2) margin or deposit to guarantee contracts, which is returned to the trader along with profit on the transaction. The "minimum" margin, P^M, was U.S.\$500. Accordingly, the opportunity cost of the margin is at least

$$\text{U.S.\$500} \cdot (91 \text{ days}/365 \text{ days}) \cdot r \ (= rP^M),$$

where r is the interest rate. It is U.S.\$25 even if the interest rate is 20 per cent. The value per contract on that day was $P^f =$ U.S.\$19,375 (= unit 25,000 pounds × U.S.\$0.7750 per pound). Thus the ratio of transaction costs to value per contract is negligible. In the final stage, elimination of unobservable expectation variable $E_{t-3}(P_t{}^s)$ puts equation (13) in a form usable for estimation:

$$P_t{}^s = a_{t-3} + (1/U_{t-3})P_{t,t-3}{}^f + e_t, \qquad (14)$$

where e_t is the forecast error defined as $P_t{}^s - E_{t-3}(P_t{}^s)$. Equation (14) is the basic equation used for estimation in the next two sections.

IV. EMPIRICAL EVIDENCE FROM THE COPPER MARKET

Equation (14) and data on monthly copper prices on the London Metals Exchange may be used to obtain estimates for U_t.

This series of monthly spot prices and three-month futures prices for copper is taken from various issues of *Metal Statistics* (Frankfurt am Main). The sample period is the 123 months from January 1970 to March 1980.

One important comment should be made before proceeding to estimation. The intercept a_{t-3} and the coefficient U_{t-3} in equation (14) are not literally constant, but generally time-variant. This can be easily seen by noting that the intercept consists of the interest rate, transaction cost, and monthly output level, all of which clearly fluctuate over time.[6] Moreover, pointed out in Section II, U_{t-3} also varies according to level of learning. Consider a case where agents initially form the subjective expectation ($P_{t,t-3}^{e}$) that is identical with mathematical expectation $E_{t-3}(P_t^s)$ and where the market is subjected to some strong sudden impact. An immediate consequence of such a shock is the divergence of subjective expectations from mathematical expectation because market participants have inadequate information about the new disturbance. However, as traders learn more about the nature of the impact and how it affects market prices, U_{t-3} approaches unity. This example well illustrates the way U_{t-3} moves over time according to the phase of the learning process.

Therefore, it is likely that using equation (14) to make an estimation over the entire sample period without paying much attention to the variability of the intercept and to U_{t-3} will give misleading results. Accurate estimates of U_{t-3} are ideally obtained by regressing P_t^s on $S_{t-3}+r_{t,t-3}P_{t-3}^M$, $Q_{t-3}Var_{t-3}(P_t^s)$, and $P_{t,t-3}^f$. However, this ideal procedure is possible only if time-series data for all transaction costs, interest rates, minimum margin rates, output level of all producers participating in the London Metals Exchange, and conditional variance of the spot price are available monthly. Unfortunately, the real situation is far from the ideal. Therefore, as a second alternative, the Kalman filter technique was used to estimate the equation. Taking the output vector as given, a Kalman filter is used to optimally estimate the static vector in a time-varying linear dynamic system.

The Kalman filter can be applied to variable parameter models such as equation (14) to obtain a series of time dependent parameter estimates.[7] The following two equations are used to express linear regression models with a varying coefficient vector β_t:

$$y_t = x_t'\beta_t + \varepsilon_t, \quad (t=1, 2, \cdots, N), \tag{15}$$

$$\beta_t = C\beta_{t-1} + \eta_t, \quad (t=1, 2, \cdots, N), \tag{16}$$

where

y_t = dependent variable,

x_t = column vector with k explanatory variables,

β_t = column vector with k coefficients,

ε_t i.i.d. $N(0, \sigma^2)$,

[6] The conditional variance of the spot price will also vary if it is assumed that the distribution of spot price shifts over time.

[7] For more information on the Kalman filter technique see [10] [2, Chap. 10]. This exposition of the Kalman filter estimation is taken mainly from [2].

η_t i.i.d. $N(0, \Omega)$,

$C = (k \times k)$ coefficients matrix.

Define $\beta_{t|s}$ as $\beta_{t|s} \equiv E(\beta_t | I_s)$ and $\sum_{t|s}$ as $\sum_{t|s} \equiv Cov(\beta_t | I_s) = E(\beta_t - \beta_{t|s})(\beta_t - \beta_{t|s})'$, where I_s is an information set consisting of $y_1, y_2, \ldots, y_s$. The estimates $\beta_{t|t}$ by the Kalman filter technique are obtained from the following set of equations:

$$\textstyle\sum_{t|t-1} = C \sum_{t-1|t-1} C' + \Omega, \tag{17}$$

$$\textstyle\sum_{t|t} = \sum_{t|t-1} - \sum_{t|t-1} x_t (x_t' \sum_{t|t-1} x_t + \sigma^2)^{-1} x_t' \sum_{t|t-1}, \tag{18}$$

$$\textstyle K_t = \sum_{t|t-1} x_t (x_t' \sum_{t|t-1} x_t + \sigma^2)^{-1}, \tag{19}$$

$$\beta_{t|t} = \beta_{t|t-1} + K_t (y_t - x_t' \beta_{t|t-1}), \tag{20}$$

$$\beta_{t|t-1} = C \beta_{t-1|t-1}. \tag{21}$$

C, Ω, and σ^2 are assumed as known for the time being. Given $\sum_{0|0}$, $\sum_{t|t}$ $(t = 1, 2, \ldots, N)$ can be calculated, in succession, from equations (17) and (18). Using the computed series of $\sum_{t|t-1}$ allows the calculation of K_t $(t = 1, 2, \ldots, N)$ from equation (19). Having obtained the series K_t, $\beta_{t|t}$ and given $\beta_{0|0}$, $(t = 1, 2, \ldots, N)$ can be computed from equations (20) and (21) in succession. Using the initial data-series subset and the generalized least squares (GLS), starting value $\beta_{0|0}$ and its conditional covariance matrix $\sum_{0|0}$ are estimated. C is taken here as the identity matrix I. Since the values of Ω and σ^2 are not known a priori, σ^2 and Ω are estimated by the maximum likelihood method.

In using the Kalman filter to estimate equation (14) the first twenty observations are set aside to obtain the GLS estimates of $\beta_{0|0}$ and $\sum_{0|0}$. Following the algorithm described, the estimated series $\beta_{t|t}$ and its conditional covariance $\sum_{t|t}$ were obtained with estimates of σ^2 and Ω. The time-varying coefficient estimates of a_t and $(1/U_t)$ thus obtained are reported on Appendix Table I.[8] Estimates between June 1971 and June 1972 are discarded because of highly unstable fluctuations. The literature points out that parameter estimates may be quite unstable for periods after the starting sample period used to obtain the estimates of $\beta_{0|0}$ and $\sum_{0|0}$. Table I gives estimates of U_t, the degree of learning in the market. Figure 10 traces the estimated movement over time of U_t and Figure 11 traces the same for the intercept a_t, both indicating considerable fluctuation.

The high variability of U_t is thought to reflect both the continuous occurrence of fluctuations in the copper market and the traders' process of learning about the shock's nature and effect. As seen above, the intercept has two parts, (1) cost incurred in making contracts for delivering commodities in the futures, expressed as the term $-(1/U_{t-3})(S_{t-3} + r_{t,t-3}P_{t-3}^M) < 0$, and (2) risk premium of the expected spot price over the futures price or degree of normal backward retardation, ex-

[8] Hypothesis testing here should be interpreted as a rough approximation of true testing, because serial correlations of error terms are not corrected. The three-month-ahead forecast error (e_t) in equation (14) is shown to follow a moving average of order two process [6]. Estimation that fails to take the presence of serially correlated errors into account, yields biased standard errors of estimators, even if the estimator is consistent, a question discussed in [8] [7] [1].

TABLE I

ESTIMATES OF DEGREE OF LEARNING IN THE MARKET (U_t)

		U_t			U_t			U_t
1972:	7	0.5803	1975:	1	1.3053	1977:	7	1.2931
	8	0.3070		2	0.8486		8	1.2839
	9	0.1871		3	0.8668		9	1.2495
	10	0.3479		4	1.1745		10	0.9608
	11	0.6048		5	1.4863		11	0.8004
	12	0.7858		6	1.3858		12	0.8837
1973:	1	0.6040		7	1.1733	1978:	1	1.0900
	2	0.4778		8	1.0804		2	1.0664
	3	0.6807		9	0.6992		3	1.0969
	4	0.8680		10	0.5027		4	0.9857
	5	0.7537		11	0.5112		5	1.0688
	6	0.6954		12	0.6504		6	1.0006
	7	0.8443	1976:	1	0.8755		7	0.9062
	8	0.7890		2	1.0487		8	0.6634
	9	0.5905		3	1.2037		9	0.6524
	10	0.5778		4	1.5449		10	0.7412
	11	0.7546		5	1.3812		11	1.1379
	12	1.1564		6	1.3854		12	1.3330
1974:	1	2.1064		7	1.0671	1979:	1	1.6325
	2	2.1882		8	1.0049		2	1.1787
	3	2.9231		9	0.8623		3	0.9379
	4	2.4331		10	1.0926		4	0.7609
	5	2.1277		11	1.2327		5	0.8603
	6	1.9497		12	1.5041		6	0.9336
	7	2.5718	1977:	1	1.5118		7	0.7850
	8	2.2400		2	1.6969		8	0.6854
	9	1.3155		3	1.4620		9	0.9702
	10	0.9242		4	1.3186		10	1.4869
	11	1.2485		5	1.2355		11	1.9066
	12	1.7784		6	1.1641		12	1.5209

Fig. 10. Movement of U_t

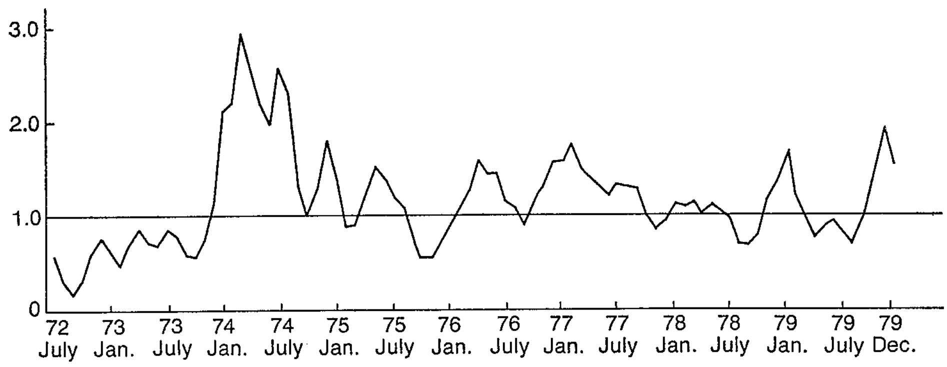
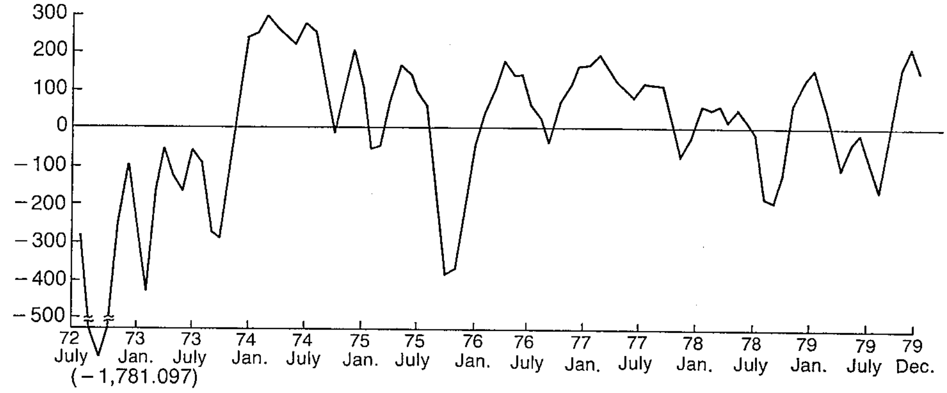


Fig. 11. Movement of a_t



pressed as the term $(1/\theta_{t-3}^2)Q_{t-3}Var_{t-3}(P_t^s) > 0$. The intercept started to rise in October 1973 when war broke out in the Middle East and OPEC unilaterally hiked its oil price 21 per cent, then hovered at quite high levels for the nine months from January to September 1974. The intercept's upward push was caused by wide oscillations in the spot price for crude oil during this period, which boosted the market demand for high risk premiums.

Figures 10 and 11 show that the intercept is co-variable with the degree of learning, which is explained by the term that represents costs incurred in making future contracts. Differentiating the definition of a_t with respect to U_t, $(\partial a_t)/(\partial U_t) = (1/U_t^2)(S_t + r_{t+3,t}P_t^M)$ shows clearly that $(\partial a_t)/(\partial U_t)$ is positive, and that explains why a_t and U_t move together.

The movement of U_t, degree of learning in the market, shows that fluctuations in U_t are closely related to events affecting the copper market. In other words, the movement of U_t can be accurately pinpointed by observing major events in copper and related markets. In the first place, U_t was kept below unity for the seventeen months from July 1972 to November 1973. The spot price for copper on the London Metals Market constantly declined by 19 per cent per annum from January 1970 to September 1972. Although the price bottomed out in September 1972 and then began to rise, the predicted spot price in the market was consistently below the mathematical expectation calculated from the objective distribution function. This may reflect traders' anticipations that the market would continue to bearish, dragged along by a long spell of price decline. It took market participants fourteen months to wipe out bearishness and revise their expectations upward to nearer true expected values.

Triggered by OPEC's sharp increase in the oil price, resources began to flow from financial assets into oil, copper, and other real assets that were believed to be inflation proof. This affected the London Metals Exchange in a significant way and in January 1974, traders quickly adjusted their expectations of the spot price far above true expectations, believing that continued heavy trading in the copper market would cause the copper price to soar for some time. The spot price on the London Metals Exchange actually peaked in January 1974. Movements in the spot price for copper hit traders so hard from September 1972 to January 1974, that the upward bias of their spot price predictions lingered on for eight months (February to September 1974) after the spot price peaked. These two episodes show that, after a long spell of price rise or fall, market participants take almost one year to correct the bias of their predictions toward true expectations.

The next event important to the level of learning occurred in November 1974, when the Council of Copper Exporting Countries (CIPEC) or an organization of copper-exporting, developing countries, reduced copper production by 10 per cent. The market responded fairly quickly and in November 1974, U_t jumped above unity to the 1.25 level. However, the expectation overshoot lasted only three months, as market participants realized that CIPEC's move was ineffective in preventing a decline in copper prices.

To stop the price fall, CIPEC raised the quota to 15 per cent of exports in April 1975. The market again reacted quickly and U_t rose to 1.17 in the same month and stayed there for five months, although traders' predictions did not diverge from the true mathematical expectations to a large extent. In fact, as bearishness prevailed in the London Metals Exchange, U_t slipped down to 0.70 in September 1975, reflecting the traders' pessimism.

In March 1977, Peru, Zaire, and Zambia agreed to reduce copper output and sales in order to maintain stable prices. Traders may have predicted this agreement three or four months before, since the market overshot its expectations around November or December 1976 and sent the U_t to 1.23 and 1.50 in those months. The mild market overshoot lasted almost one year, then subdued with U_t starting to oscillate around unity in October 1977.

On November 1, 1979, OPEC raised its per barrel price for crude oil 33 per cent from U.S.$18.00 to U.S.$24.00. The copper market responded one month before the OPEC announcement and revised the prediction of the future spot price upward above true expected value in anticipation of a coming speculative boom in the commodity markets.

In general, market participants quickly respond to events in copper and related markets and revise their expectations swiftly, incorporating the consequences of events into the copper price. Revised predictions of future spot prices tend to overshoot. The degree of overshoot caused by the insufficient knowledge of the nature of the events, quickly disappears and the subjective expectation converges to the objective, mathematical one once the market participants know that the events are transitory. However, if the event turns out to be permanent, the process of learning is slow. Even after the effects of the event dissipate completely, its legacy remains a memory with traders for about one year and affects the way in which they form expectations of future spot prices.

Spectral analysis of data obtained in Table I gives the thirty-month cycle shown in Figure 12 and Table II.

Fig. 12. Spectral Analysis

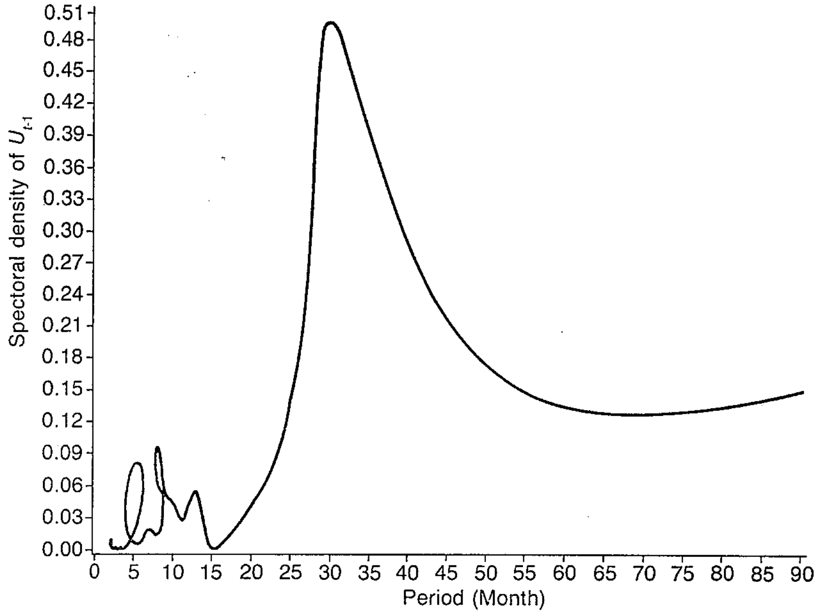


TABLE II

SPECTRAL ANALYSIS

Period (Month)	Spectral Density[a]	Period (Month)	Spectral Density[a]
More than 90.0	0.149832	3.9	0.000025
90.0	0.149832	3.7	0.000847
45.0	0.218828	3.6	0.000390
30.0	0.497306	3.4	0.000193
22.5	0.072504	3.3	0.002799
18.0	0.020691	3.2	0.000184
15.0	0.001010	3.1	0.001240
12.8	0.054761	3.0	0.000195
11.2	0.028540	2.9	0.000663
10.0	0.044226	2.8	0.001017
9.0	0.050402	2.7	0.002364
8.1	0.098024	2.6	0.000678
7.5	0.016221	2.5	0.000983
6.9	0.020279	2.5	0.000163
6.4	0.014383	2.4	0.001516
6.0	0.005944	2.3	0.001201
5.6	0.004763	2.3	0.000439
5.2	0.084705	2.2	0.001489
5.0	0.010820	2.1	0.001446
4.7	0.007345	2.1	0.000691
4.5	0.014427	2.0	0.007856
4.2	0.004363	2.0	0.001338
4.0	0.001977	2.0	0.003214

[a] Log transformation value.

V. EMPIRICAL EVIDENCE II

The entire sample period was divided into five subperiods[9] based on the results of Table I in Section IV, and estimates were made for each period using the following equation:

$$P_{t,t-3}{}^{f} = U_{t-3}(P_t{}^{s}) + a_{t-3}{}' - U_{t-3}e_t, \tag{22}$$

where $a_{t-3}{}' = -U_{t-3}a_{t-3}$.

A careful choice of criteria used to divide the entire sample period into its parts will justify this procedure and make it possible to obtain relatively stable estimates of the intercept and U_t. The criteria used here are based upon important events in the world copper market that have affected the intercept and U_t significantly.

Subperiod I is the thirty-three months from January 1970 to September 1972. Subperiod II is the seventeen months from October 1972 to February 1974, the period of U.S. copper export quotas. Subperiod III lasts twenty-one months from March 1974 to November 1975. Subperiod IV lasts for the twenty-nine months that started in December 1975 with CIPEC's decision to reduce output by 15 per cent and ends in April 1978. Subperiod V is the twenty-nine months from May 1978 to March 1980 and is characterized by changes in U.S. producer pricing policies. Specifically, Kennecott Copper announced in May 1978 a price for copper that would be the closing spot price of the previous COMEX (the New York Commodity Exchange) trading session plus 2.5 cents per pound.[10] Equation (22) was estimated for each subperiod by the OLS method. The estimated results are listed in Table III. U_{t-3} is not different from unity at the 5 per cent significance for subperiod I.[11] This result implies no significant deviation of subjective expectation from mathematical expectations that are attributable to sufficient levels of learning. For subperiods II, IV, and V, the sizes of U_t are less than one ($U_t < 1$). The null hypothesis for the intercept, $a_{t-3}{}' = 0$, is not rejected at 5 per cent significance for subperiods I, II, and III.

9 A three-month time lag makes subperiod I the thirty months from April 1970 to September 1972.

10 Immediately after that announcement, the wide gap between the U.S. producers price and the price on the London Metals Exchange vanished as both parties began to coordinate their pricing efforts.

11 Hypothesis testing here should also be interpreted as a rough approximation of true testing. That is because the statistical regression package generally reports incorrect standard errors of estimates in the presence of serially correlated errors.

Note that the composite error term $-U_{t-3}e_t$ in equation (22) is serially correlated because forecast error e_t correlates with e_{t-1} and e_{t-2} even though it does not correlate with e_{t-3}, e_{t-4}, ... In fact, the low *D.W.* statistics in Table III indicates a strong positive serial correlation. Correction of this anomaly by the Cochrare-Orcutt method is not justified here, for the reason that the three-month-in-advance forecast error follows a moving average of the order two process [MA(2)], but not an AR(1) or AR(2) process. See [6]. For further discussion of this issue and alternative estimation procedures, see [8] [7].

TABLE III

THE ESTIMATED VALUES FOR U_t OF THE COPPER PRICES

	Sample Period	U_t	a_{t-3}'	R^2	D.W.	t-statistics for $U_t=1$
I.	Jan. 1970–Sept. 1972	0.9869 (7.8)	36.4745 (0.69)	0.66	0.48	−0.10
II.	Oct. 1972–Feb. 1974	0.7113 (11.34)	86.5898 (1.59)	0.90	1.45	−4.60*
III.	Mar. 1974–Nov. 1975	1.4859 (6.20)	−203.2585 (−1.36)	0.67	0.78	2.03***
IV.	Dec. 1975–Apr. 1978	0.5338 (2.43)	362.5904 (2.17)	0.18	0.28	−2.10***
V.	May 1978–Mar. 1980	0.3801 (1.69)	558.5873 (2.68)	0.12	0.43	−2.76**

Notes: 1. Parentheses show t-values.
2. *, **, and *** means that the null hypothesis ($U_t=1$) is rejected at the significance level of 0.005, 0.01, and 0.025 respectively.

U_t's value during subperiod I shows that the hypothesis of rational expectations holds quite well during that period. Conventional models of expectations, such as adaptive or auto-regressive expectations, based on price history, are often said to underestimate the true mathematical expectations that are conditional on current information when traders forecast events that significantly alter copper prices. The reason for this assertion is that price history does not convey information about future events, but, in fact market participants do actually form their expectations according to that information. In such situations, evaluations of buffer stocking policy based upon commodity models, with expectations modeled conventionally or nonrationally, are likely to misinform the buffer authority about size and frequency of market intervention. This misinformation results from an incorrect specification of U_t as less than unity, although the true U_t does not differ from unity. The upshot is that during subperiod I, the size and frequency of interventions in the copper market are smaller than when calculated by econometric copper models that express expectations as a function of past prices only.

Learning in the market for subperiods II, IV, and V is not yet complete and the magnitude of U_t is less than one. This implies that evaluations of price stabilization policy by copper models that form expectations nonrationally may provide authorities with fairly precise information on the intervention patterns of buffer stock operations. Subperiod III, with its U_t magnitude of 1.49 would be a particularly interesting subject for further study.

Estimates of copper U_t on the London Metals Exchange took widely varying values in each subperiod. That probably reflects both a continuous sharp impact on the copper market and the traders' process of learning about the nature and effects of those impacts.

VI. CONCLUSION

The hypothesis of catastrophic expectations emphasizes the role of learning in the process of forming expectations. This hypothesis indicates that agents form subjective expectations that are identical with rational expectations only when they have a sufficiently high level of learning. Empirically demonstration of a learning process in the copper market gives evidence with important implications for policies to stabilize prices in primary commodity markets. In examining comparative characteristics of market intervention patterns under different expectation schemes, Ogawa [13] emphasizes how crucial expectations are to evaluating price stabilization policies. For a broad class of policy rules free of excess capital gains or losses in phases of market price change, the hypothesis of rational expectations predicts fewer and smaller market interventions, and therefore, smaller financial requirements of policy, than do nonrational expectation models. Market intervention patterns and costs incurred in the process of market operations are fairly sensitive to the magnitude of U_t, the degree of learning in the market.

It is often argued that when traders anticipate events significantly affecting future market prices, conventional models of expectations based on past prices, such as adaptive expectations models, tend to underestimate true mathematical expectations that are conditional on current information. The reason for that assertion is that past prices do not contain information about future events. However, market participants actually use that information to form their expectations. Evaluating market intervention policy with a commodity model, in which expectations are derived conventionally or irrationally, will likely misinform policy authorities about the size and frequency of the market intervention needed. That misinformation is the result of an incorrectly specified U_t. The upshot is that during a period when U_t is unity, the size and frequency of market interventions are smaller than those calculated in an econometric model with expectations expressed as a function of past prices only. This implies that policy authorities should have accurate information about market conditions, such as the presence of factors disturbing market prices and the speed with which traders accommodate to these factors before intervening in the market. Such information is necessary to efficiently stabilize commodity prices at their lowest levels.

These findings are crucial to the problem of price stabilization that is discussed by Turnovsky [15]. Turnovsky says that Massell's demonstration that stabilization provides net gain to both producers and consumers still holds for both rational and adaptive expectations [15, p. 131].

There are, however, two different phases: (a) when U_t is less than one and (b) when U_t is greater than one. Unlike Turnovsky [15], Kuchiki [11] finds that total gain from stabilization is negative when the U_t phase is greater than one. Considering U_t's thirty-month cycle, price stabilization would not be desirable when the U_t phase is greater than one.

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APPENDIX TABLE I

ESTIMATES OF TIME-VARYING COEFFICIENTS

		a_t	$(1/U_t)$			a_t	$(1/U_t)$
1972:	7	−281.467**	1.7231**		7	271.946**	0.3888**
		(−4.48)	(12.04)			(13.37)	(11.83)
	8	−916.166**	3.2569**		8	248.104**	0.4464**
		(−13.67)	(21.31)			(12.41)	(14.04)
	9	−1,781.097**	5.3453**		9	118.284**	0.7602**
		(−33.98)	(45.62)			(4.76)	(17.56)
	10	−57.123**	2.8742**		10	−14.856	1.0870**
		(−27.44)	(50.60)			(−0.47)	(18.17)
	11	−251.698**	1.6536**		11	101.489**	0.8010**
		(−12.38)	(42.53)			(3.47)	(15.00)
	12	−93.940**	1.2726**		12	200.311*	0.5623**
		(−6.80)	(55.89)			(7.71)	(12.40)
1973:	1	−252.567**	1.6558**	1975:	1	115.942**	0.7661**
		(−19.22)	(79.74)			(4.52)	(17.28)
	2	−433.496**	2.0929**		2	−54.701	1.1784**
		(−29.69)	(87.46)			(−0.92)	(23.07)
	3	−175.214**	1.4690**		3	−44.464	1.1537**
		(−13.63)	(75.82)			(−1.40)	(19.65)
	4	−43.991**	1.1520**		4	80.704**	0.8515**
		(−4.12)	(83.74)			(3.03)	(18.48)
	5	−116.338**	1.3268*		5	154.686**	0.6728**
		(−11.04)	(101.25)			(6.81)	(18.51)
	6	−162.346**	1.4379**		6	134.485**	0.7216**
		(−14.54)	(100.51)			(5.42)	(17.51)
	7	−57.418**	1.1845**		7	80.383**	0.8523**
		(−5.29)	(89.42)			(3.12)	(19.72)
	8	−91.758**	1.2674**		8	50.046	0.9256**
		(−8.72)	(104.36)			(1.93)	(21.24)
	9	−268.099**	1.6934**		9	−158.851**	1.4303**
		(−24.49)	(131.53)			(−5.90)	(31.27)
	10	−283.530**	1.7307**		10	−390.286**	1.9893**
		(−26.59)	(145.45)			(−15.35)	(47.51)
	11	−115.694**	1.3253**		11	−376.543**	1.9561**
		(−11.46)	(129.36)			(−15.29)	(49.25)
	12	74.940**	0.8648**		12	−203.221**	1.5376**
		(7.80)	(98.32)			(−10.12)	(53.83)
1974:	1	236.386**	0.4747**	1976:	1	−39.478*	1.1422**
		(25.37)	(60.78)			(−2.41)	(58.58)
	2	243.740**	0.4570**		2	38.598*	0.9536**
		(25.14)	(54.06)			(2.39)	(50.91)
	3	291.287**	0.3421**		3	89.436**	0.8307**
		(26.89)	(31.28)			(5.73)	(48.39)
	4	262.771**	0.4110**		4	165.373**	0.6473**
		(20.25)	(25.93)			(10.93)	(40.94)
	5	238.357**	0.4700**		5	133.629**	0.7240**
		(17.33)	(26.89)			(8.28)	(40.05)
	6	220.600*	0.5129**		6	134.533**	0.7218**
		(12.05)	(18.17)			(8.12)	(38.20)

APPENDIX TABLE I (Continued)

		a_t	$(1/U_t)$			a_t	$(1/U_t)$
	7	45.431*	0.9371**		4	13.477	1.0145**
		(2.55)	(43.24)			(0.56)	(30.71)
	8	21.414	0.9951**		5	46.152*	0.9356**
		(1.19)	(44.97)			(2.00)	(30.43)
	9	−46.729**	1.1597**		6	19.725	0.9994**
		(−2.51)	(49.88)			(0.85)	(32.38)
	10	54.504**	0.9152**		7	−23.366	1.1035**
		(3.08)	(43.88)			(−1.03)	(37.46)
	11	97.548**	0.8112**		8	−190.549**	1.5074**
		(5.57)	(39.95)			(−8.33)	(50.71)
	12	158.147**	0.6648**		9	−201.088**	1.5329**
		(9.37)	(35.85)			(−8.96)	(53.77)
1977:	1	159.551**	0.6615**		10	−125.062**	1.3493**
		(8.87)	(31.39)			(−5.95)	(54.08)
	2	189.406**	0.5893**		11	69.731**	0.8788**
		(9.89)	(24.88)			(3.76)	(46.61)
	3	150.215**	0.6840**		12	122.977**	0.7502**
		(7.51)	(26.78)			(6.75)	(42.04)
	4	119.436**	0.7584**	1979:	1	179.943**	0.6126**
		(5.57)	(26.29)			(9.79)	(33.86)
	5	98.324**	0.8094**		2	82.348**	0.8484**
		(4.04)	(22.68)			(4.22)	(41.05)
	6	77.760**	0.8591**		3	−7.802	1.0662**
		(3.30)	(25.51)			(−0.38)	(47.20)
	7	113.257**	0.7733**		4	−110.427**	1.3142**
		(4.73)	(22.46)			(−4.94)	(48.20)
	8	110.949**	0.7789**		5	−47.610*	1.1624**
		(4.26)	(19.82)			(−2.27)	(48.91)
	9	102.073**	0.8003**		6	−9.7907	1.0711**
		(4.19)	(22.79)			(−0.49)	(51.08)
	10	2.552	1.0408**		7	−93.747**	1.2739**
		(0.10)	(26.34)			(−4.79)	(63.44)
	11	−83.778**	1.2494**		8	−170.397**	1.4591**
		(−2.97)	(28.32)			(−8.76)	(74.24)
	12	−35.001	1.1316**		9	6.9313	1.0307**
		(−1.34)	(29.03)			(0.36)	(53.77)
1978:	1	53.668**	0.9174**		10	155.213**	0.6725**
		(2.21)	(26.89)			(8.74)	(44.00)
	2	45.255	0.9378**		11	216.503**	0.5245**
		(1.94)	(29.58)			(12.75)	(39.59)
	3	56.072*	0.9116**		12	161.449**	0.6575**
		(2.43)	(29.50)			(8.51)	(36.75)

Note: Figures in parentheses are ratios of coefficient estimates to standard errors.
* indicates significance at 5 per cent.
** indicates significance at 1 per cent.