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AN ECONOMIC ANALYSIS OF GROUP LENDING PROGRAMS IN DEVELOPING COUNTRIES

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I. INTRODUCTION

GROUP lending programs became popular in many developing countries during the 1970s. For example, the number of organized groups in Ghana grew from 23 in 1969 to over 5,000 in 1980.[1] Similar, but less dramatic growth was observed for groups in the Dominican Republic, India, Malawi, Mexico, Nepal, the Philippines, and Zimbabwe. The often cited rationale for these loans is to provide credit to farmers who would otherwise not receive loans, primarily, it is claimed, because the transaction costs of individual loans are too high. In many instances, however, the rapid growth of group loans, due in large part to subsidized credit, moved beyond the economic margin of profitability. Thus, only a few of these programs are regarded as successful, where success is measured by low default rates and access to credit for small borrowers.[2] Many programs have experienced wholesale increases in arrears and high delinquency rates, thereby jeopardizing the lending program and the viability of the institutions underwriting the loans.

The goal of this paper is to provide an economic analysis of group lending.[3] The characteristics of successful lending programs will be described. We will argue that a successful program brings together individuals who share a common economic interest apart from the loan. The successful lending contract will recognize the external factors affecting the group and incorporate, perhaps indirectly, these factors into the loan agreement. The loan contract may then become self-enforcing under most economic scenarios. For example, joint liability is an important feature of many group loan contracts. This clause is useful if it is enforceable and if the members of the group gain individually by the misfortune of another member or if the other members avoid a cost from repaying another member's loan. The element of individual gain or cost avoidance is the external

[1] Owusu and Tetteh [13] provide a summary of the group loan experience in Ghana, including data on loans made, groups formed, and membership size.

[2] Braverman and Guasch [7] have argued that subsidized credit does not generally reach small borrowers and that it tends to increase the informal market loan interest rate, thereby becoming a net cost to the small borrower.

[3] Group lending can be expected to increase agricultural output. In effect, it reduces the costs of inputs. Farmers no longer have to save at low or negative rates before they can finance investment projects. In addition, the availability of credit may hasten the adoption of new technologies. It will also lead to important changes in the economic organization of the rural sector. However, these questions are outside the scope of this paper.

factor exploited by the joint liability clause. When this element is not present, the default of one member tends to encourage the group as a whole to default, thus exacerbating the delinquency problem. Unsuccessful programs tend to lack a common factor among group members or have not fully exploited it in the loan agreement.

A frequent argument made for the use of group loans is that they lower the transaction costs of banks [1] [5].[4] Economies of scale in lending may make banks more willing to lend to groups rather than to individuals. It should be noted, however, that group loans transfer the transaction costs from the banks to the group. The group must allocate loans, evaluate collateral, and monitor members' performance. This transfer of costs is often overlooked when claims are made that group loans lower transaction costs. At present, no data is available on the extent of the transaction costs of groups.

Moreover, the transaction cost–savings, if any, of a group loan must be set against the cost of organizing groups. Most lending programs form groups by sending loan solicitors into rural areas. These solicitors are trained to organize farmers into a group. They instruct potential members on how to apply for loans, methods of coordinating production, the need for regular group meetings, generally weekly, and the consequences of default. They are expected to act like loan officers for the group until the group can elect their own representatives and self-manage the loan. This process may take two or three years resulting in significant organizational costs.

We argue that the transaction cost explanation cannot explain the rapid growth of groups or their subsequently high default rates. It may offer a reason for forming groups by some marginal borrowers, who may be denied loans on an individual basis. That is, when the collateral and success rates of individual borrowers are pooled for a single, group loan, the loan becomes economically viable ceteris paribus, because the loan transaction costs have fallen sufficiently to justify making the loan. The long-term viability of a group and likelihood of repayment, however, will depend more on the economic relationship between members of the group than the transaction cost–savings between the group and the bank. In the simplest of terms, the transaction cost explanation is the supply-side argument for making group loans, while the economic interrelationships between group members provides the demand-side explanation for group formation and stability, which may be affected by the terms of the loan contract.

The most important factor stimulating the growth of group loans is the availability of subsidized credit from government and international donor sources.[5]

[4] Saito and Villanueva [14] document the transaction costs of some financial institutions in the Philippines. While their figures are not for group loans, per se, they indicate that transaction costs for small-scale agricultural loans are nearly three times greater than for large-scale industry loans. The difference is observed in both administrative and default expenses. It is reasonable to expect that agricultural group loans reduce these costs. They also document the relatively high level of arrears in many credit institutions servicing LDCs.

[5] While this claim cannot be proven unambiguously, the tendency for these programs to fail when credit rates increase suggests that many groups are formed to gain access to cheap credit. Von Pischke and Adams [17], Vogel and Larson [16], Adams [1], and others have commented on the effects of cheap credit on rural financial markets.

Before many of these programs began, group lending by private banks was on a much smaller scale, and primarily occurred through cooperative credit institutions, which acted as lender to and borrower from the same individuals. This analysis will not focus on the subsidized nature of these loans, but rather concentrate on how program success or failure is affected by program and loan contract specifications.

In the next section, we analyze the economic rationale for group loans, pointing out the features of groups that make their members interdependent and thus more likely to repay their loans. Emphasis is placed on the organizational assets and liabilities of groups; that is, the assets and liabilities created by the group that are not present in individual loan contracts. Transaciton costs are also examined in more detail. The characteristics of successful group loans are described. Section III develops a model of the lending process. The basic model is a version of the competitive market model developed by Besanko and Thakor [4]. This model is motivated by an information asymmetry between the lender and borrower. There are high- and low-risk borrowers in the loan market that are indistinguishable to the lender. The lender's problem is to design a loan contract that maximizes profit and reveals who is high or low risk. This model is extended to encompass group lending and the case of imperfect monitoring of project outcomes. The individual loan model is reexamined with the joint liability clause to study the effects of group loans. Loan collateral is used in this formulation to illustrate the economic interdependence between group members. It is assumed that the group repossesses the collateral of a defaulting member. The repossessed collateral is more valuable to group members than to the bank or other outsiders, thus group members gain more than the bank if another member defaults. The results are notably different than with the individual loan problem. Both high- and low-risk members are required to post collateral, whereas with individual loans only low-risk individuals are required to post collateral. The effects of increasing the opportunity cost of loanable funds, however, are similar between the two formulations. With imperfect monitoring it is assumed that banks must audit borrowers to determine whether a project has succeeded, which is not necessary for group loans. A Nash equilibrium game is constructed to determine the optimal strategy for lenders and borrowers. The results show that imperfect monitoring raises the lender's opportunity cost of funds, which increases loan charges to both high- and low-risk borrowers. Finally, section IV offers a few concluding remarks.

II. GROUP CHARACTERISTICS AND INCENTIVES

Groups that are potential candidates for loans may be organized before a loan solicitor approaches a rural area. There are many aspects of rural production that encourage cooperation or association between farmers or other small businessmen. The economic environment in many rural areas in LDCs is not unlike that found in some of the early New England settlemens. These settlements established a system of rules and penalties to facilitate life in general and economic activity in particular. On the former level, there were rules to provide for law

and order, and volunteers available to apprehend and administer justice to those who violated these rules. Punishment ranged from an economic or social penalty, usually on the grain harvested by the offender or public display in a pillory, to banishment from the village. Other rules, such as taxing a resident of the settlement if one of their relatives, who may have newly arrived from a less prosperous community, becomes a vagrant or does not help with the harvest of the settlement, also imposed an economic penalty.

These early settlers recognized the economic costs of externalities. They organized production for the village in the early years of the settlement to realize scale economies that were not available to individuals. They also recognized the incentive problems of common ownership and production, and thus as time passed they afford individuals who were more productive the opportunity to purchase parcels of land. Labor services were taxed in different ways to aid the common goals of the settlement. The battlements surrounding many villages or the assembly/refuge buildings were maintained on a rotating or cooperative basis by the various residents, drinking water was often collected by a cooperative effort, and monitoring of children was generally the responsibility of the entire settlement.

Some of these elements generating external costs and benefits to the early New England settlers may also provide incentives to form groups in rural areas of LDCs. For example, landowners with contiguous parcels of land face problems of irrigation and drainage that encourage cooperation or laws defining each farmer's property rights. Such laws might exist, and riparian rights are an example, but they may not be enforceable in rural areas. Thus, farmers organize to resolve the conflicts that may arise between them. A loan solicitor may utilize these external factors to make a successful group loan.

A. *Organizational Assets and Liabilities*

The various institutions granting group loans have generally recognized the need for a bond among group members beyond the loan itself. For example, the Agricultural Development Bank of Nepal, which has administered the Small Farmers' Development Program (SFDP) since 1975, has provided loans to more than two thousand groups.[6] They send solicitors, known as group organizers, into an area to conduct a pre-investment survey. The purpose of this survey is to define the pattern of production and income in an area. With this information, they are encouraged to organize farmers who own *contiguous* parcels of land and are of *similar* economic status. These informal groups usually consist of between ten to fifteen members.

The desire to select relatively homogeneous individuals with adjacent landholdings for a group implicitly recognizes the value of an external influence on group members. Farmers with similar landholdings and income may be expected to have the same probability of succeeding in a loan project. If they had significantly different chances of success, their incomes and landholdings would have diverged over time. By using the homogeneity rule, the SFDP and other lenders are reducing

[6] Mosley and Dahal [12] discuss the origin and implementation of the SFDP, including an interim assessment of its performance.

the potential for cross-subsidization between group members. Cross-subsidization is possible because groups generally accept some form of joint liability (see below). Group members with a low chance of success may be subsidized by those with a greater success rate. If the potential members know as much about each other as the pre-investment survey supposedly reveals to the group organizer, they would not be willing to accept joint liability, ceteris paribus, with noncomparable members *if* they intended to repay the loan.[7] If groups are organized with non-homogeneous members, which might occur if some members misrepresent their economic status, then the potential for default or delinquency is high and the chance that the group will remain together over time is low. *The first feature then of a successful group loan is that it is made to fairly homogeneous individuals.*

Identifying homogeneous individuals for group membership may be a time-consuming process. Undoubtedly, one of the factors contributing to the SFDP's increasing rate of delinquency[8] is the unrealistic time horizon imposed on group organizers. According to Mosley and Dahal [12, p. 198], group organizers are "expected to obtain detailed information on family members, land-holdings, subsidiary occupations and income, agricultural occupations with details of crops and types of livestock plus income per household and per capita, individual skills, and purposes for which the loan is required, for an *entire panchayat* (about 1,000 families, or 6,000–7,000 people) within the space of one month" (emphasis in original). With these information requirements and the one month time constraint, it is likely that group organizers will make many mistakes in defining homogeneous groups.

Classifying potential members by their landholdings may minimize some of these errors, particularly if potential members own contiguous parcels, which is a screening rule suggested by the SFDP. Farmers with similar size landholdings probably generate similar incomes, and thus landholdings may be a reasonable proxy for project success. The selection of contiguous landowners serves two additional purposes, both of which will be exploited by the joint liability feature of most group loans. First, contiguous landowners may observe each other's efforts. Thus the cost of monitoring the success or failure of group members is lower. This feature may thus serve to lower the default costs of group loans.

The second reason for selecting continguous landowners is that they may value the other members' parcels higher than the valuation of other landowners or the lending agency. Contiguous parcels may be cultivated and integrated into the harvest cycle at lower cost than noncontiguous parcels. Contiguous landowners will understand more parcel-specific details—such as irrigation, drainage charac-

[7] It is likely that some groups have no intention of honoring their joint liability agreement with the lender. For these groups, homogeneity is not important because the potential for cross-subsidies to other group members is nonexistent. These groups are motivated by the "cheap" credit that is offered by lenders. Homogeneity of group members may become important, however, if the lender denies access to "cheap" credit to groups with outstanding arrears, which is often the policy of lenders participating in group loan programs.

[8] Mosley and Dahal report that "within the scheme [SFDP] as a whole, the amount overdue increased 65 percent in the 1982/3 financial year" [12, p. 199].

teristics, and soil fertility—than landowners who have not been afforded the opportunity to observe the crop cycle on a piece of land firsthand.[9] Hypothetically speaking, these individuals would be the highest bidders in an auction for the property owned by another group member if that member could not repay his share of the group loan. Thus, contiguous landowners may be willing to accept the liability of other members because they may gain valuable property in the event of default. This assumes, of course, that all group members post some form of collateral[10] to the group, which may be claimed and divided among all non-defaulting members in the event a member defaults on his loan.[11] Dispersed-property owners will gain less by the default of a group member, and will be less willing to accept the joint liability clause or, more realistically, less willing to accept responsibility for another member's loan after default has occurred, which will encourage default by the entire group.

Not all group loans are made for crop production. Some loans are for livestock development or cottage industries, such as leather crafts, weaving, and indigenous art products. The former projects are much like crop production in their relationship to the land. Contiguous landholdings may bond the group members because of the convenience afforded the livestock. They may be easily monitored and fed from contiguous parcels of land. Therefore, contiguous landowners involved in livestock projects will have a relatively high valuation of each other's parcels. These observations suggest that the relative valuation of parcels by group members is partly related to the type of project undertaken.

With cottage industry projects, however, group members may not benefit from ownership of contiguous parcels of land.[12] The project itself may not produce external benefits directly valued by contiguous landowners. To recreate a situation

[9] Offsetting these benefits somewhat is the fact that owners of contiguous parcels of land are not well-diversified. For example, a flood may destroy the entire crop on several contiguous parcels of land. If the parcels were more dispersed, some of them may escape the effects of a flood, leaving the landowner with some income-producing assets. Undoubtedly, diversification is valuable when unexpected events, such as a flood, occur. The costs of diversification, however, are also significant. If a farmer must travel between parcels on a daily basis to irrigate or tend the soil, protect the crop from pests, or harvest crops, then the time spent traveling must be taken into account. If floods or other natural disasters are infrequent events, the costs of diversification will outweigh its benefits and the arguments made below will be consistent.

[10] As group lending schemes are applied in widely varying countries with very different social and economic systems, the type of collateral in these schemes differs from country to country. But in most countries collateral consists of land or livestock. In addition, the larger the loan obtained by the individual the greater is the collateral posted. Usually, the collateral is greater than the value of the loan. But in general the larger the collateral posted by the individual the greater is the responsibility for delinquent loans.

[11] Burkett [9] has noted that there may be political difficulties with enforcing such an agreement, because it would tend to redistribute assets from the less fortunate to the more fortunate borrowers.

[12] The only tangible benefit to contiguous parcels for cottage industry loans is the lower cost of monitoring each group member's effort. This is probably not a significant factor, however, because the output of a member is generally observable on a regular basis, when the goods are marketed, which occurs more frequently than crop harvests or livestock sales.

where there is a difference in relative valuations, each participant in a cottage industry project must offer some group-specific capital as collateral. An example may be the tools used to craft leather goods. These tools are generally more valuable to an individual with leather-crafting skills (i.e., another member of the group) than to someone without such skills.

The group-specific capital may be intangible in some instances. The investment made in monitoring the performance of other group members is an example of intangible group-specific capital. This investment will increase the likelihood that all group members succeed in repaying their loans. Individual members will encourage other members to work in earnest to reduce the likelihood that the more productive members subsidize the less productive members via the joint liability clause. Monitoring of group members is largely unique to the habits of each member, that is, whether the member irrigates regularly, searches for and controls for an infestation of pests, fertilizes crops, etc., and may be completely unknown until the group has experienced a crop cycle for the first time. The investment in monitoring and understanding the practices of other group members will be lost if the group dissolves itself. To the extent that each member undertakes an investment in monitoring or other intangible group-specific capital, the group will have a common bond that tends to encourage repayment. *The second feature then of a successful group loan is that some group-specific capital is involved, which is usually satisfied if members own contiguous parcels of land but may be satisfied by an intangible investment that is depreciated if the group dissolves itself.*

Membership size is another factor influencing the success of a group loan. Generally, lending agencies encourage group organizers to find between ten to fifteen members for a group loan. This range is also small enough to allow contiguous landownership. One of the major reasons why the Dominican Republic group lending project, sponsored by the Dominican Development Foundation (DDF), had mixed success with joint liability of group members is that it consistently loaned to groups whose average size exceeded this range. The DDF groups generally involved twenty or more members.[13] The DDF program may have fared better if the liability for repayment of the group loan resided with one or two group leaders, as in the case of the program in Ghana, where the average group size was fifty for nearly ten years and group leaders were held jointly and severally liable for repayment of the loan.[14] *Although not independent of the form of group liability, the third feature of a successful group loan is relatively small membership in each group.*

Joint liability is used in nearly all group lending programs. It may be quasi-voluntary or mandatory liability and may affect all group members or only the group leaders. Bratton [6] reports on the success of a quasi-voluntary liability

[13] Adams and Pablo Romero [2] report on the progress of DDF lending from 1966 to 1979. Information is provided for average loan size, the number of groups, new loans made, and membership size by fiscal year.

[14] Unfortunately for the Ghana program, fifty members appears to be too difficult to monitor by only one or two leaders. Thus, the effect of large and diffuse membership overwhelms the efficiency of making only a few group leaders liable. Repayment rates averaged only 85 per cent in this program according to Owusu and Tetteh [13].

scheme in Zimbabwe. Under this scheme, group members were not jointly liable for the default of another member, but the entire group was denied further credit until the bad debt was paid. Instead of benefiting from the default of another member by repossessing his land or other assets, group members bear a cost from another member's default. Bratton reports that the repayment rates under this approach are less than for mandatory liability (an average of 72 per cent versus 85 per cent), except when there is a drought and then quasi-voluntary liability works better (18 per cent versus 9 per cent). The reason for this finding is not clear, because mandatory liability is expected to dominate quasi-voluntary liability under all conditions. It may be that the loans made under the quasi-voluntary liability program were to farmers less affected by the Zimbabwe drought of 1982–83, but Bratton offers no evidence for or against this hypothesis.

Usually, joint liability is assigned to all group members. For this approach to be effective, the members must benefit in some way from repaying another member's loan. If there is group-specific capital pledged as collateral by each member, then there is a tangible benefit to joint liability. On the other hand, without such group-specific capital, either tangible or intangible, the default of one member encourages the default of another, because repaying another member's loan is like a tax, which may be avoided by default. The probability of default then on a group loan does not follow a binomial distribution, assuming independence between members, but rather is equal to the probability of failure of the most unskilled or unlucky member.[15] This observation alone may explain why default rates are relatively high in group loan programs.

Groups that are formed solely for the purpose of gaining access to cheap credit epitcmize this problem. There is no bond between group members. If one defaults, there is no incentive to accept the burden of repayment by the other members unless, say, further access to subsidized credit is denied. One may argue then that members may benefit from repayment if they avoid a cost in the process, although subsidized credit does not appear to be an "efficient" mechanism for encouraging repayment.[16] The quasi-voluntary liability program in Zimbabwe is one example where access to credit is used to encourage repayment. Members may avoid the cost of being excluded from future loans by paying another member's debt. This is effective when the loans are made at subsidized rates and the cost of repaying the debt does not offset the value of the difference between the free market interest rate and the subsidized interest rate. Without a subsidized interest rate, this threat is ineffective.

Moreover, the threat to withdraw subsidized credit is expected to be self-fulfilling at some point in the future. The destiny of all subsidized loan programs

[15] If we assume that the probability of success of each group member is fixed for the period of the loan and independent of another member's chance of success, then the binomial distribution describes the likelihood that a given number of loans will fail. This distribution is developed in Section III and incorporated in the principal-agent model of lending. An excellent reference to the binomial distribution is Larsen and Marx [11, p. 54].

[16] An argument in favor of such a carrot and stick scheme implicitly values sunk costs. Many government projects, after they have been found to be uneconomic, receive more funds because, it is claimed, if the project is stopped, all that has been invested is lost. Using access to subsidized credit to encourage repayment involves the same sort of reasoning.

is to dissolve when the subsidy becomes too great a burden or when access to the program is restricted to maintain its subsidized nature. In either case, rational groups will expect a day of reckoning when the likelihood of receiving a loan is significantly less than unity. Immediately preceding this point in time, groups that are held together only by their desire for cheap credit will find it advantageous to renege on their loans. If there are enough "cheap credit" groups with this incentive, the loan program may collapse or restrict membership before the day of reckoning. As the day of reckoning moves closer, the behavior of "cheap credit" groups is affected as they recognize that the incentive to default is greater. This process may continue to a point where the threat to withdraw access to cheap credit is ineffective in preventing default.[17]

If group membership becomes large, the value of joint liability in encouraging members to monitor and assist other, floundering, members is reduced. Repayment rates, ex post, may be low as marginal members default. A domino-like effect may sweep through the group if there are enough marginal members, encouraging default by the entire group. To encourage monitoring and instruction for marginal members, some lending programs have assigned joint liability to a few group leaders. As a percentage of their wealth, these individuals will be significantly affected by even one default, thus they will invest more time in preventing such an event. Joint liability for all members then is effective for small groups. After groups reach a certain critical mass, it may be better to specialize both the administrative and liability functions of the group. If, however, groups become too big, even the specialization of group liability may be ineffective, as too many members are too difficult and costly to monitor and, generally, the only threat available after a loan is made is the withdrawal of future loans. *The fourth feature then of a successful group loan is some form of joint liability, preferably for all members of a small group.*

A feature that may be more successful than joint liability in ensuring repayment is the use of margins or compensating balances on group loans. Schaefer-Kehnert [15] reports that group loans in Malawi required a security fund of 10 per cent of the loan amount for each member. If there is a default, the fund is used for repayment and must be replenished before new loans are made. Schaefer-Kehnert reports that this program experienced repayment rates of 99–100 per cent between 1972 and 1979. While it is not possible to attribute these repayment rates solely to the security fund, because other factors such as joint liability are involved here, it does appear that the security fund is important. The other programs that have used joint and several liability have experienced repayment rates of between 15–85 per cent. By contrast, using a security account along with joint liability generates nearly a perfect repayment record. *The fifth feature then of a successful group loan is the use of a security account that is drawn on to cover defaults and replenished before new credit is granted.*

[17] This problem is similar to the problem arising when a central bank offers foreign reserves at discount prices. A "speculative" attack on the foreign reserves of a central bank may be expected when such a policy is followed.

B. *Transaction Costs*

It has been suggested that loan processing has significant scale economies. Braverman and Guasch [7] claim that processing costs for small loans may range from 15 per cent to 40 per cent of the loan value. These costs may be divided into administrative and collection costs. On the administrative side, banks may find it less costly to make group loans than individual loans, but then the groups must bear the costs of managing these loans. Seen in this light, the economies of scale argument is questionable. Banks may find it less costly to process a few group loans for a given sum than many individual loans for the same amount, but now there are many different groups performing the monitoring, evaluation, allocation, and collection services. The overhead or fixed costs of these services are duplicated across these groups, so total administrative costs—summed over the banks and the various groups—may increase. The empirical question then is whether collection costs are lower on group versus individual loans.

The relatively poor performance of group loans, however, makes it likely that both types of loans have high collection costs. Banks and other lenders may have the same difficulty collecting arrears on group loans as on individual loans if the reason for forming the groups was simply to receive a subsidized loan. As noted above, groups that are motivated by the availability of cheap credit or lack some form of group-specific capital may default *in total* if a single member defaults. The collection costs then are higher as if an equivalent number of individual loans had been made in the first place.

It might also be noted that diversification of risk is a common argument made for preferring group to individual loans. While this is not a transaction costs argument, it is sometimes mentioned in the same breath as transaction costs, and thus deserves attention. The claim is that group loans lower the variance of a lender's loan portfolio and therefore expose a lender to less risk than an individual loan. This is an apples and oranges comparison. If the lender had made the same number of individual loans as there are members of a given group, the portfolio's variance would be the same as the case of a group loan. There is no change in the relative risk to a lender from a group loan.

III. A PRINCIPAL-AGENT MODEL OF LENDING

The basic issue to be modeled is: When do lenders desire to lend to individuals or groups and when do individuals desire to borrow funds by themselves or within a group. This problem will be addressed in an environment of asymmetric information. Lenders, even those using group organizers, generally know less about the expected payoff from a project than borrowers. In a formal sense, this uncertainty may manifest itself in two ways. The lender is unable to accurately estimate either the probability of project success, the potential payoff, or both. The lender is thus confronted with incomplete information about borrowers. Besanko and Thakor [4] develop a credit market model where lenders must decide what interest rate to charge, the amount of collateral required, and whether to

make the loan in an environment with high- and low-risk borrowers. The lender cannot distinguish between these two types of borrowers ex ante. The problem then is to design a loan contract that maximizes profits while separating high- and low-risk borrowers. The revelation principle implies that the profits produced by a truth-revealing contract are no less than those generated by a loan contract where agents misrepresent their risk status. A simplified version of this model is used as a starting point for our analysis of group loan contracts. This version develops the loan contract for an individual loan and serves as the basis of reference for the group loan contract and the case of imperfect monitoring.

A. *Basic Model*

It is assumed that lenders and borrowers are risk neutral, so that none of our results are driven by an aversion to risk by one party or the other.[18] Each borrower has a known end-of-period endowment, W, part or all of which may be pledged as collateral, C, for a loan. The collateral may be invested in the project, but it is assumed not to depreciate regardless of the project outcome. The investor may request a loan from the lender and invest the proceeds in a risky project, which has a payoff of R if it is successful and zero otherwise. The lender observes the success or failure of the project without cost. This assumption eliminates the moral hazard problem of the borrower misrepresenting the project outcome to the lender.

The probability that a risky project succeeds is δ, which varies with the riskiness of the borrower. The lender faces a pool of observationally identical borrowers, who are either high- or low-risk types. A high-risk borrower will succeed with probability δ_1. The low-risk borrower will succeed with probability δ_2, where $\delta_2 > \delta_1$. There are γ borrowers who are high-risk and $1 - \gamma$ low-risk borrowers in the loan pool.

The lender's problem is to design two loan contracts that will maximize profits and provide borrowers with an incentive to self-select by level of risk; that is, the high-risk borrowers will select one contract and the low-risk borrowers the other. These contracts are thus *incentive compatible* from the borrowers' viewpoint. The lender specifies the probability (π_i) of granting credit on each contract, the amount of the loan plus interest (α_i), and the amount of collateral required (C_i). The index i (= 1, 2) denotes the two loan contracts and will also correspond to whether the contract is intended for the high- or low-risk borrower, respectively. The lender must pay an amount, r, for the deposits necessary to make a loan.

[18] In contrast, Braverman and Guasch [7] develop a principal-agent model of the lending process where borrowers are risk averse. Their analysis may be criticized for separating loan proceeds and income from the sale of the output in the utility function. Both variables represent income that may be used to purchase goods. There does not appear to be a meaningful reason to separate them. In addition, they assume that joint liability in a group loan implies that each member receives the average output of the group as his reward. This serves to lower the variance of a member's income and thus may provide an incentive for risk averse individuals to join a group. However, the more productive group members will not accept such a scheme, thereby creating an incentive for the group to dissolve. Moreover, there does not appear to be any evidence that groups follow this practice.

There is a disparity between the collateral valuation of the lender and the borrower. The lender values collateral at a fraction (β) of the borrower's assessment. This is reasonable because the lender in many rural projects may not be in a position to capitalize fully the value of a borrower's parcel of land (the usual form of collateral). The cost of managing the property, particularly for small parcels, may be significant for a lender operating from a distant city.

All lenders are assumed to operate in a perfectly competitive credit market. Excess profits are expected to be zero. We will examine the Nash equilibrium loan contracts in this environment.[19] With asymmetric information, the equilibrium contracts will maximize the surplus of both high- and low-risk borrowers subject to the incentive compatibility constraints, a zero profit constraint for lenders, and the usual technical restrictions. Formally, the problem may be stated as follows.

$$\text{Maximize } \gamma\pi_1[\delta_1(R-\alpha_1)-(1-\delta_1)C_1] + (1-\gamma)\pi_2[\delta_2(R-\alpha_2)-(1-\delta_2)C_2],$$

subject to:

$$\pi_1[\delta_1(R-\alpha_1)-(1-\delta_1)C_1] \geq \pi_2[\delta_1(R-\alpha_2)-(1-\delta_1)C_2], \quad (1a)$$

$$\pi_2[\delta_2(R-\alpha_2)-(1-\delta_2)C_2] \geq \pi_1[\delta_2(R-\alpha_1)-(1-\delta_2)C_1], \quad (1b)$$

$$\delta_i\alpha_i+(1-\delta_i)\beta C_i=r, \quad (2)$$

$$0\leq\pi_i\leq 1, \quad (3)$$

$$0\leq C_i\leq W, \quad (4)$$

where $i=1,2$.

The solution to this problem depends on whether endowment is a binding constraint on collateral requirements. When endowment is not a binding constraint the equilibrium loan contracts are defined as follows.[20]

$$\alpha_1=r/\delta_1,\ C_1=0,\ \pi_1=1, \quad (5)$$

$$\alpha_2=(r/\delta_2)-\beta(1-\delta_2)(C_2/\delta_2),\ C_2=r(\delta_2-\delta_1)/[\delta_2(1-\delta_1)-\beta\delta_1(1-\delta_2)],\ \pi_2=1. \quad (6)$$

At this solution, the contract selected by the low-risk borrowers has a lower cost of funds ($\alpha_2<\alpha_1$) than that selected by the high-risk borrowers. To prevent high-risk borrowers from preferring the second contract, the lender requires collateral from borrowers desiring the more favorable contract terms. Collateral acts as a sorting mechanism in this model. It imposes a greater expected cost on the high-risk borrower because these borrowers are more likely to lose their collateral. They will therefore find the first contract more profitable because it does not require collateral.

It may also be noted that this solution may be used when there is a difference in endowments between borrowers. The high-risk borrowers may be expected to have less endowment than the low-risk borrowers, because low-risk borrowers will be more successful over time. The lender may then set collateral requirements

[19] Besanko and Thakor [4] provide the rationale for focusing on Nash equilibrium contracts.

[20] This is Proposition 2 in Besanko and Thakor [4, p. 676] subject only to slight modification. The proof follows the outline of their proof, which is given in the Appendix of their paper.

on the more favorable loan just above the endowment of the high-risk borrower. This strategy will effectively sort borrowers by risk. High-risk borrowers are not rationed because they may apply for the more expensive loan contract.[21]

There are only a few comparative static results for this solution. If the cost of funds (r) to the lender increases, loan interest rates increase for high- and low-risk borrowers proportional to their respective probabilities of success. Collateral requirements for low-risk borrowers also increase. If the collateral becomes more marketable, so that β increases, borrowing costs and collateral requirements on low-risk loans will decrease. A comparison between more or less risky borrowers is inherent in the problem, and thus omitted.

B. *Group Loans with Joint Liability*

The lender's problem with a group loan is quite different from an individual loan. Group loans with joint liability potentially offer the lender a lower-risk loan. To include the concept of joint liability in the problem, it is assumed that the group will fully indemnify the lender against default. Thus, the lender will not worry about a group default.[22] The individual is concerned about joint liability, however, because it must offer some tangible gain over an individual loan or the group membership will not be valuable. We assume that the members of the group who repay a defaulting member's loan will benefit by sharing that member's collateral. Collateral to a group member is valued at market prices. There is no penalty placed on the value of collateral as was the case with the individual loan problem.

The probability of success or failure is the same across group members. That is, high- and low-risk borrowers identify each other and exclude one another from group membership if the risk type of an applicant does not match that of the group. This is only sensible, because low-risk borrowers would find themselves subsidizing high-risk borrowers if group membership was open to all applicants.[23]

[21] Rationing may occur in this model if the collateral requirements in equation (6) exceed the end-of-period endowment of the low-risk borrowers. Proposition 3 in Besanko and Thakor [4, p. 678] addresses this situation. The solution for the low-risk borrower changes to require that all endowment is posted as collateral and that the lender randomize its loan decision. The probability of receiving a loan is less than unity for low-risk borrowers. The probability terms now act as a sorting device in the contract. Unfortunately, this means that some low-risk borrowers will be denied loans to encourage the high-risk borrowers to apply for the more expensive loan contract. This does not appear to be a very realistic solution.

[22] The problem of group default may be modeled in a manner similar to that of the risk-free group loan. The objective function of the borrower is modified to make the probability of default depend on the number of other members in default. The binomial part of the objective function then has two parts. One that represents the expected gain from only a "few" defaults by other members and another that represents the net gain (or loss) from the group defaulting. The number of members who must default before there is a group default is endogenous to the model.

[23] Unfortunately, there are no detailed studies of group lending schemes which allow us to provide empirical evidence on this question. In particular, we have no evidence on whether groups select themselves by risk class. However, it would appear that the greater the riskiness of the projects as a whole the higher the collateral required.

The banks' dilemma is to distinguish between high- and low-risk groups. The competitive equilibrium solution solves the following problem.

$$\text{Maximize} \quad \gamma\pi_1 g(\delta_1, \alpha_1, C_1)+(1-\gamma)\pi_2 g(\delta_2, \alpha_2, C_2),$$

subject to:

$$\pi_1 g(\delta_1, \alpha_1, C_1)\geq\pi_2 g(\delta_1, \alpha_2, C_2), \tag{7a}$$

$$\pi_2 g(\delta_2, \alpha_2, C_2)\geq\pi_1 g(\delta_2, \alpha_1, C_1), \tag{7b}$$

$$\delta_i\alpha_i=r, \tag{8}$$

$$0\leq\pi_i\leq 1, \tag{9}$$

$$0\leq C_i\leq W, \tag{10}$$

$$g(\delta_i, \alpha_i, C_i)=\delta_i(R-\alpha_i)-(1-\delta_i)C_i+\sum_{a=0}^{N} B(N, a, \delta_i)[a/(N+1-a)](C_i-\alpha_i), \tag{11}$$

where $i = 1, 2$, $B(N, a, \delta_i)$ is the binomial distribution function for the joint probability of group failure, and the number of group members equals $N + 1$. The binomial distribution function is defined as $B(N, a, \delta) = \{N!/[(N - a)!a!]\}\delta^{N-a}\cdot(1 - \delta)^a$. The term in brackets following the binomial distribution function is the share of liability assessed on a representative, non-defaulting member of the group. The share increases as the number of defaults (a) increases. A group member must pay his share of the loan obligation, but receives his share of the defaulting member's collateral. Essentially, equation (11) is a borrower's expected gain from an individual loan plus the expected net gain from accepting joint liability.

Notice that the lender's profit function, equation (8), does not include any collateral. This is because the lender is assumed to be fully indemnified against default. If this were not the case, the lender's profit would include a term similar to the binomial distribution term in equation (11).

The solution to the group loan problem is summarized as follows:

$$\alpha_1=r/\delta_1, \quad C_1=W, \quad \pi_1=1, \tag{12}$$

$$\alpha_2=r/\delta_2, \quad C_2=W+[(\alpha_2-\alpha_1)(\delta_1+Q)]/[Q-(1-\delta_1)], \quad \pi_2=1, \tag{13}$$

where $Q=\sum_{a=0}^{N} B(N, a, \delta_1)[a/(N+1-a)]$.

It may be shown that $Q-(1-\delta_1)>0$.[24] Because the high-risk group pays more than the low-risk group, $\alpha_2<\alpha_1$, the collateral required from low-risk individuals is less than their endowment (i.e., $C_2<C_1=W$). This situation is markedly different from the individual loan contract solution. Collateral is required from low- and high-risk individuals receiving group loans, whereas only low-risk individuals are required to post collateral for individual loans. Moreover, the collateral required of high-risk individuals is greater than that for low-risk individuals. The intuition for these results is that the joint liability clause creates an opportunity for gain and loss by group members. The members of a high-risk group will share the burden of repaying another member's debts more frequently than members of

[24] This proof is shown in the Appendix.

a low-risk group. They will therefore require more collateral to offset this burden.

If the lender's cost of funds increases, the effect on loan charges is the same as before; they will increase. The effect on collateral requirements, however, are different. Collateral required on the high-risk loan contract will not change—it is already at the maximum amount—but collateral required on the low-risk contract will decrease, thereby partly offsetting the higher loan costs. The intuition for this result is straightforward. Both the high- and low-risk groups face higher loan charges. The high-risk group cannot offset the higher loan charges by increasing collateral requirements, which would provide group members with a larger return if another member defaults, therefore their expected profits will fall. To discourage high-risk groups from applying for low-risk contracts, the collateral requirements are lowered on these contracts. The low-risk individuals will not benefit as much as before when there is a default, so their expected profits will fall too.

Long-term group stability requires that the expected profits from a group loan with joint liability exceed the expected profits from an individual loan. In the case of a high-risk individual, this requires that $W[Q-(1-\delta_1)] > \alpha_1 Q$. The term on the left-hand side of the inequality is the gain from accepting a group loan, measured as the product of the collateral required and the difference between the probability of other group members defaulting and the probability of the individual defaulting. This gain must exceed the expected cost of the other members' defaults, which is the right-hand-side term. A similar, although significantly more complicated, condition may be defined for low-risk individuals.

It may be noted that if groups are formed solely to gain access to subsidized credit, the joint liability formulation is incorrect. Assuming that subsidized credit is available only once, then the probability of default on a group loan is equal to the probability of default on an individual loan. All group members will default if a single, unluck member defaults. If subsidized credit is available on an ongoing basis, group default may be discouraged by withholding new credit to groups with any payments in arrears.[25] The analysis of this problem is similar to the above model in most respects, except that an additional gain is received by the members who repay a bad loan. This may be easily added to the binomial component of the profit function and the results are similar to the above.

C. *Imperfect Monitoring*

In the individual loan model above, it is implicitly assumed that the lender may determine the outcome of the project without incurring any cost of monitoring. However, if this is not possible, it may be profitable for the borrower to misrepresent the project outcome and surrender the collateral. This is clearly the case with high-risk borrowers who post no collateral. A high-risk individual may

[25] Allen [3] develops a model where the threat of exclusion from the credit market is sufficient to encourage repayment of a loan. His model assumes infinite-lived agents. It is not clear whether his analysis will work with finite-lived agents, because they may find it in their interest to default in the final period, which may cause the lender to withhold the loan in this period. The agent will then default in the next to lasted period etc., and the model breaks down as no loans are made unless some collateral is involved.

gain R and lose nothing, assuming the lender believes he is truthful, by claiming the project failed when, in fact, it was a success. If this occurs frequently enough, some form of project audit will then develop to attempt to discover the actual project outcome.

To incorporate this possibility into our model, we will assume that the lender and borrower solve a bi-matrix, single period game. The solution to this game will determine whether monitoring or cheating occurs. The game formulation allows the lender and borrower to recognize the interdependence of their actions. For example, if the lender monitors every project, the borrower will not cheat. But if monitoring is costly and the agent does not cheat, there is no return to monitoring. The lender then has an incentive to monitor less, which provides the borrower with an incentive to cheat.

The behavior of the lender will be represented by the strategy, y, which represents the probability that the project will be audited. Similarly, the strategy of the borrower, x, represents the probability that the borrower will cheat by misrepresenting the project outcome. It is assumed that monitoring a project requires A dollars per audit. An audit will determine cheating with probability $0 < m < 1$. If the lender detects cheating, the agent is fined an amount equal to F.[26] The effects of these assumptions on the expected profit of the lender and borrower are shown in the following expression (the subscript for high- and low-risk borrowers has been omitted for notational convenience).

Borrower:

$$\begin{aligned} E(\pi) = {} & [\delta(R-\alpha) - (1-\delta)C](1-x)(1-y) \\ & + (\delta R - C)x(1-y) + \{[\delta(R-\alpha) - (1-\delta)C - F]m \\ & + (\delta R - C)(1-m)\}xy + [\delta(R-\alpha) - (1-\delta)C](1-x)y. \end{aligned}$$

Lender:

$$\begin{aligned} E(\pi) = {} & [\delta\alpha + (1-\delta)\beta C - r](1-x)(1-y) + (\beta C - r)x(1-y) \\ & + \{[\delta\alpha + (1-\delta)\beta C - r - A + F]m + (\beta C - r - A)(1-m)\}xy \\ & + [\delta\alpha + (1-\delta)\beta C - r - A](1-x)y. \end{aligned}$$

The object of the borrower and lender is to select a value for x and y that maximizes profits, respectively. The Nash equilibrium of this game admits three pure strategy solutions and a mixed strategy solution. The pure strategy solutions are endpoint strategies. The first pure strategy solution is that the agent does not cheat and the principal does not monitor. This occurs when the repayment on the loan (α) is less than the required collateral (C). By cheating in this situation, the borrower loses assets more valuable than the loan repayment. This is probably why many loans are offered as a percentage of the value of the collateral. In the individual loan model, this solution is unlikely because the lender must devise a contract to sort between the high- and low-risk borrowers. The second pure

[26] The problem may be solved with or without a fine. The basic conclusions remain the same. The only major difference is that the lender will always audit in the mixed strategy solution without the fine. That is, the lack of a fine encourages the borrower to cheat, which can only be prevented by monitoring.

strategy solution is that the borrower cheats but the lender does not monitor. This occurs when the cost of monitoring (A) exceeds the expected gain from monitoring, $[\delta(\alpha - \beta C) + F]m$. The last pure strategy solution is that the borrower cheats and the lender monitors every project. The only reason this occurs is that the agent's gain from cheating offsets any expected losses due to monitoring. This solution may be eliminated by an appropriately large fine.

We will focus on the mixed strategy solution. At this solution, the borrower cheats part of the time, while the lender monitors some of the projects. The frequency of cheating and monitoring are given by the following expressions.

$$x^* = A/[\delta(\alpha - \beta C) + F]m, \tag{14}$$

$$y^* = \delta(\alpha - C)/[\delta(\alpha - C) + F]m. \tag{15}$$

At the mixed strategy solution, the borrower's profits are equal to $\delta(R - \alpha) - (1 - \delta)C$, which is the same as the profit function for the individual loan problem with perfect monitoring. The reason that the borrowers profits are not increased is that the lender—at the margin—will have an incentive to monitor more frequently if the borrower gains from cheating. Thus, monitoring will drive profits back to their "no cheating" level. The borrower's profits cannot be less than this amount because the borrower has the *option* to cheat, and thus will never be placed at a disadvantage by his own option.

The lender's profits, however, will be reduced to reflect the costs of monitoring and the losses incurred due to undetected cheating. At the mixed strategy solution, the lender's profit function is given by the following expression:

$$E(\pi) = \delta\alpha + (1 - \delta)\beta C - r - \delta(\alpha - \beta C)x^*, \tag{16}$$

which differs from equation (2) only by the last term. This term measures the lender's losses from cheating and monitoring.

Substituting equation (16) for equation (2) in the individual loan problem provides the proper representation of the imperfect monitoring problem. The solution to the imperfect monitoring problem, however, does not lend itself to a simple, closed form solution, primarily because both α and C are found in the denominator of x^*. As long as $\alpha > \beta C$, which holds for the individual loan solution, then the imperfect monitoring problem effectively increases the opportunity cost of funds to the lender, because $\delta(\alpha - \beta C)x^* > 0$. This implies that the stated borrowing costs (α) are higher under the imperfect monitoring problem than under either the individual or group loan problems. These observations suggest that a group loan would be preferred from the lender's and borrower's point of view, assuming that the lender is indemnified against losses with a group loan.

IV. CONCLUSIONS

Group loans are an interesting financial innovation in developing countries. The loan contracts for these loans have generally contained economically sensible clauses. We have outlined the motivation behind these clauses and shown where

mistakes may occur. This analysis has produced five requirements for a successful group loan contract:

(1) Group members are homogeneous with respect to economic criteria;
(2) Some group-specific capital is created by the members of the group or by the loan contract. Usually contiguous landholdings will satisfy this requirement;
(3) Group membership is small enough to be consistent with the group-specific capital requirements;
(4) Joint liability is assigned either to all members of the group or a few group leaders;
(5) A security or margin account is specified in the loan contract to cover any defaults.

In addition, we have developed a formal model of group lending with joint liability. The model supports our observations about successful group loan contracts. The model shows that collateral by group members, which may be group-specific collateral, will act to bind the members to each other. If members gain from the default of another member—by sharing the member's collateral—they will accept joint liability and may prefer a group loan to an individual loan contract. The results from this model are different from the individual loan model developed by Besanko and Thakor [4] and others.[27] We find that collateral is required for both high- and low-risk individuals requesting a group loan, whereas the individual loan model only requires collateral from low-risk individuals. The group loan model implies that high-risk individuals are the first rationed if their endowment is insufficient to meet collateral requirements, whereas the individual loan model suggests that low-risk individuals are rationed. The results for the imperfect monitoring problem complement the group loan and individual loan results. With imperfect monitoring, the lender will face a higher opportunity cost of funds, which leads to a higher loan interest rate for both high- and low-risk borrowers when compared to either the individual or group loan problems.

[27] Chan and Kanatas [10] show that if a lender's valuation of collateral is equal to the borrower's valuation, then no collateral will be required in the loan contract. Our results for group loans show that collateral is required even when the lender is not the beneficiary of such collateral.

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APPENDIX

SOLUTION TO THE GROUP LOAN PROBLEM

Following Besanko and Thakor [4], we begin by assuming that equation (7b) is slack and that equation (7a) is binding. Thus, for C_1 and C_2 we have equation (A.1):

$$\begin{aligned}\text{Maximize } \pounds = {} & \gamma\pi_1[\delta_1(R-\alpha_1)-(1-\delta_1)C_1+Q_1(C_1-\alpha_1)] \\ & +(1-\gamma)\pi_2[\delta_2(R-\alpha_2)-(1-\delta_2)C_2+Q_2(C_2-\alpha_2)], \qquad \text{(A.1)}\end{aligned}$$

subject to equation (7a) in the main text.

Using $\alpha_i = r/\delta_i$ from equation (8), we can ignore the optimal conditions for α_i. Differentiating with regard to C_i gives:

$$\frac{\partial \pounds}{\partial C_1} = \gamma\pi_1[-(1-\delta_1)+Q_1]+\mu\pi_1[-(1-\delta_1)+Q_1] \lesseqgtr 0, \qquad \text{(A.2)}$$

$$\frac{\partial \pounds}{\partial C_2}=(1-\gamma)\pi_2[-(1-\delta_2)+Q_2]-\mu\pi_2[-(1-\delta_1)+Q_1]\leqslant 0, \tag{A.3}$$

where μ is the multiplier for equation (7a) and Q_i is the binomial distribution term evaluated at δ_i. Note that if $Q_i-(1-\delta_i)>0$ then $C_1=W$ because $\partial \pounds/\partial C_1>0$. The solution for C_2 is ambiguous because the terms are opposite in signs. So the first step is to show that $Q_i>1-\delta_i$. For arbitrary δ_i, the problem is stated as:

$$\sum_{a=0}^{N}\frac{N!}{(N-a)!a!}\delta^{N-a}(1-\delta)^a\left(\frac{a}{N+1-a}\right)\leqslant 1-\delta. \tag{A.4}$$

Let us first examine the factorial term and the coefficient $(a/[N+1-a])$:

$$\frac{N!}{(N-a)!a!}\left(\frac{a}{N+1-a}\right)=\frac{N!}{(N-a)!(N+1-a)}\left(\frac{a}{a!}\right),$$

after some rearranging we obtain:

$$\frac{N!}{[N-(a-1)]!(a-1)!}.$$

So the proposition changes to

$$\sum_{a=1}^{N}\frac{N!}{[N-(a-1)]!(a-1)!}\delta^{N-a}(1-\delta)^a\leqslant 1-\delta.$$

Note that we start at $a=1$ because this is a zero term in the above expression. We can rewrite this expression as:

$$\sum_{a=1}^{N}\frac{N!}{[N-(a-1)]!(a-1)!}\delta^{N-(a-1)}(1-\delta)^{a-1}\left(\frac{1-\delta}{\delta}\right)\leqslant 1-\delta.$$

The term on the left-hand side of $(1-\delta)/\delta$ is equal to one by the binomial theorem as long as $\delta+(1-\delta)$ is equal to one. This holds true by assumption. So the proposition simplifies to:

$$\frac{1-\delta}{\delta}\leqslant 1-\delta \quad \text{or} \quad \frac{1}{\delta}\leqslant 1.$$

The proposition becomes trivial as $0<\delta<1$. Thus, $Q_i>1-\delta_i$ for any positive δ_i that is less than one.

Returning to equations (A.2) and (A.3), we see that $C_1=W$ because equation (A.2) is always positive. The solution for C_2 may be found by treating equation (7a) as a binding constraint (as assumed). It remains to be shown that equation (7b) is slack, which follows from simple substitution. The remaining solutions—$\pi_1=\pi_2=1$—are found by following the proof of Proposition 2 in Besanko and Thakor [4] and so are not repeated here.